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PHYSICS

1969

SovietRxiv

Abstract

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UDC 533.951.8

PHYSICS

A. A. IVANOV, E. Z. MEILIKHOV, V. V. PARAIL,
D. A. FRANK-KAMENETSKII

EXCITATION OF NONPOTENTIAL OSCILLATIONS IN THE PLASMA OF A SOLID

(Presented by Academician M. A. Leontovich, 11 XII 1968)

Recently a number of works have appeared in which various mechanisms were considered for the excitation by a current of high-frequency (10–100 MHz) oscillations in the plasma of a solid in the presence of an external magnetic field (both parallel to the current and perpendicular to it). The extensive class of instabilities considered in the works of L. E. Gurevich and I. V. Ioffe ([1]) is associated with the excitation of potential oscillations, for which the inhomogeneity of the plasma is of primary importance. The frequencies of oscillations of this type are usually not large, since the wavelengths are then of the order of the dimensions of the specimen. In works ([2,3]) the excitation of nonpotential oscillations of the helicon type was considered, with a characteristic threshold value of the drift velocity of the order of the Alfvén velocity. A paper ([4]) is devoted to the same question. However, in all these works the intrinsic magnetic field of the current was not taken into account. Below it will be shown that, although it is small in comparison with the external magnetic field, it nevertheless substantially changes the dispersion relation and the excitation conditions.

Let us consider, in the hydrodynamic approximation, the problem of excitation of nonpotential oscillations arising when a current is passed across an external magnetic field. Let us direct the z axis along the constant external magnetic field $\mathbf{H}_0$, and the y axis along the unperturbed current $\mathbf{j}_0$. In this case, in a system bounded in x (and unbounded in y and z), there arises the intrinsic magnetic field of the current, for which $\partial H_z/\partial x \neq 0$. Bearing in mind low-frequency nonpotential oscillations (with frequencies ω much smaller than the collision frequency ν), we neglect the inertia of electrons and holes in the equations of motion. Then the equations take the form

$$-en\mathbf{E} - en\frac{1}{c}[\mathbf{v}_n\mathbf{H}] - \chi T_n \nabla n - \nu n \mathbf{v}_n m_n = 0; \quad (1)$$

$$ep\mathbf{E} + ep\frac{1}{c}[\mathbf{v}_p\mathbf{H}] - \chi T_p \nabla p - \nu p \mathbf{v}_p m_p = 0. \quad (2)$$

Here n, p are the concentrations of electrons and holes; $\mathbf{v}_n, \mathbf{v}_p$ are their velocities; χ is Boltzmann' s constant. Solving these equations together with the continuity equations shows that there exists a stationary drift of the electron-hole plasma along the x axis in crossed electric and magnetic fields, while the current, under the conditions

$$\omega_{Hn} \gg \nu \gg \omega_{Hp} \tag{3}$$

(the electrons are magnetized, while the holes are not) and $n_0 \gg p_0$, is carried by electrons moving with velocity $u = j_0/n_0 e$. Here n_0, p_0 are equilibrium concentrations.

Passing to a coordinate system in which the drift of the plasma as a whole is absent, and assuming that the perturbed quantities (denoted below by the subscript 1) are proportional to $\exp(-i\omega t)$, we obtain from (1) and (2), taking (3) into account,

$$-\frac{\nu m_n}{e}\mathbf{j}^1 - \nu n^1 m_n \mathbf{u} = -en_0\mathbf{E}^1 + \frac{1}{c}[\mathbf{j}_0\mathbf{H}^1] + \frac{1}{c}[\mathbf{j}^1\mathbf{H}_0] + en^1\frac{1}{c}[\mathbf{u}\mathbf{H}_0] - \chi T_n \nabla n^1 - ep_0\frac{1}{c}[\mathbf{v}_p^1\mathbf{H}_0] - \nu m_n \nu_p p_0; \tag{4}$$

$$ep_0\mathbf{E}^1 - \chi T_p \nabla p^1 - p_0 \nu m_p \mathbf{v}_p^1 = 0, \tag{5}$$

where $\mathbf{j} = e(p\mathbf{v}_p - n\mathbf{v}_n)$.

Taking into account the quasineutrality condition $n^1 = p^1$, it is sufficient to use the continuity equation for holes

$$-i\omega p^1 + \operatorname{div}(p_0\mathbf{v}_p^1) = -\nu_R p^1, \tag{6}$$

where ν_R is the hole recombination frequency.

We shall also use Maxwell' s equations, in which we neglect the displacement current:

$$\operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\mathbf{j}; \tag{7}$$

$$\operatorname{rot}\mathbf{E}^1 = \frac{i\omega}{c}\mathbf{H}^1; \tag{8}$$

$$\operatorname{div}\mathbf{H}^1 = 0. \tag{9}$$

For what follows it is convenient to express the components $\mathbf{v}_p^1$ through $\mathbf{H}^1$ and p^1. Substituting (7) into (4), we find

$$-\frac{\nu m_n c}{4\pi e}\,\mathrm{rot}_z\,\mathbf{H}^1 - \frac{1}{4\pi}[\mathrm{rot}\,\mathbf{H}_0, \mathbf{H}^1]_z + \chi T_n \frac{\partial p^1}{\partial z} + \nu p_0 m_p v_{pz}^1 = -e n_0 E_z^1,$$

which, using (5), gives

$$v_{pz}^1 = \frac{e}{m_p \nu}\frac{\partial H_0/\partial x}{4\pi n_0 e} H_x^1 - \frac{v_{Tp}^2}{\nu n_0}\frac{\partial p^1}{\partial z} + \frac{c}{4\pi n_0 e}\frac{m_n}{m_p}\left(\frac{\partial H_y^1}{\partial x} - \frac{\partial H_x^1}{\partial y}\right). \tag{10}$$

Applying the operation rot to equation (5) and assuming $p_0 = \mathrm{const}$ (below it will be shown how the dispersion relation is modified for $p_0 = p_0(x)$), we obtain, using (8):

$$\mathrm{rot}\,\mathbf{v}_p^1 = i\frac{e\omega}{\nu c m_p}\mathbf{H}^1. \tag{11}$$

From (7) it follows that $H_0 = H_0(x)$; therefore we choose the spatial dependence of the perturbations in the form $A(x)\exp(ik_y y + ik_z z)$. Noting that, according to (7), $\partial H_0/\partial x = 4\pi n_0 e u/c$, we then obtain from (10) and (11):

$$v_{px}^1 = \frac{e\omega}{m_p \nu c k_z} H_y^1 - i\frac{eu}{m_p \nu c k_z}\frac{\partial H_x^1}{\partial x} - \frac{v_{Tp}^2}{\nu n_0}\frac{\partial p^1}{\partial x} - i\frac{1}{k_z}\frac{c}{4\pi n_0 e}\frac{m_n}{m_p}\left(\frac{\partial^2 H_y^1}{\partial x^2} - ik_y\frac{\partial H_x^1}{\partial x}\right); \tag{12}$$

$$v_{py}^1 = -\frac{\tilde{\omega}}{m_p \nu c k_z} H_x^1 - ik_y\frac{v_{Tp}^2}{\nu n_0}p^1 + \frac{k_y}{k_z}\frac{c}{4\pi n_0 e}\frac{m_n}{m_p}\left(\frac{\partial H_y^1}{\partial x} - ik_{yH}x^1\right), \tag{13}$$

where $\tilde{\omega} = \omega - k_{yu}$; $\quad v_{Tp}^2 = \dfrac{\chi}{m_p}\left(T_n\dfrac{n_0}{p_0} + T_p\right)$.

Thus we have expressed $\mathbf{v}_p^1$ and $\mathbf{E}^1$ from equation (5) in terms of $\mathbf{H}^1$ and p^1. Substituting the relations obtained into the x- and y-components of equation (4) and into equation (6), and using (9), we find the dispersion relation in the quasiclassical approximation $(\partial/\partial x \sim ik_x,\ k_x \gg \partial \ln H_0/\partial x)$

$$k^2 k_z^2 + k^4\frac{m_n p_0}{m_p n_0}\frac{\tilde{\omega} + i\nu_R}{\omega + i\nu_R + ik^2 v_{Tp}^2/\nu} - ik^2\frac{\omega_p^2}{c^2\nu}\frac{\tilde{\omega}(\tilde{\omega} + i\nu_R)}{\omega + i\nu_R + ik^2 v_{Tp}^2/\nu} = \frac{\omega_n^4}{c^4}\frac{\omega\tilde{\omega}}{\omega_{Hn}^2}. \tag{14}$$

Here $\omega_{0n}^2 = 4\pi n_0 e^2/m_n$; $\omega_{0p}^2 = 4\pi p_0 e^2/m_p$. It is important to note that the dispersion relation obtained for nonpotential oscillations is valid for $k^2 \ll k_{\max} = \omega_{0n}^2/c^2$ (5).

Equation (14) describes the natural oscillations of an electron-hole plasma of the magnetic-sound type ($k_z = 0,\ u = 0$) and of a helicon ($k_z > \sqrt{m_n p_0/m_p n_0},\ u = 0$) (5). For $p_0 = p_0(x)$ the general form of the dispersion relation is preserved, with ω in its last term replaced by $\omega - k_y u\, \partial \ln p_0/\partial \ln H_0$. Let us examine at what velocities u the oscillations described by dispersion relation (14) begin to build up. For this we use the fact that at the stability boundary $\operatorname{Im}\omega = 0$.

Equating the imaginary and real parts of dispersion relation (14) to zero, we obtain the following system of equations (ω , u are the values at the stability boundary):

$$A\omega + B\tilde{\omega} + C\frac{\nu_R}{\nu}\tilde{\omega} - D\omega^2\tilde{\omega} = 0; \tag{15}$$

$$A(\nu_R + k^2 v_{Tp}^2/\nu) + B\nu_R - C\tilde{\omega}^2/\nu - D(\nu_R + k^2 v_{Tp}^2/\nu)\tilde{\omega}\,\omega = 0, \tag{16}$$

where

$$A = k^2 k_z^2; \quad B = k^4 \frac{m_n p_0}{m_p n_0}; \quad C = k^2 \frac{\omega_{0p}^2}{c^2}; \quad D = \frac{\omega_{0n}^4}{c^4}\frac{1}{\omega_{Hn}^2}.$$

Substituting A from equation (16) into equation (15), we obtain

$$\frac{C}{\nu}\left(\nu_R\tilde{\omega} + \frac{\tilde{\omega}^2\omega}{\nu_R + k^2 v_{Tp}^2/\nu}\right) + B\left(\tilde{\omega} - \frac{\omega\,\nu_R}{\nu_R + k^2 v_{Tp}^2/\nu}\right) = 0.$$

Introduce the effective recombination frequency $\nu = \nu_R + k^2 v_{Tp}^2/\nu$. It is determined by both recombination and diffusion (v_{Tp}^2/ν is the diffusion coefficient). Then from (16), taking into account the inequalities $C \gg D\nu\nu$, $C\nu_R/\nu \ll B$, it follows that $\tilde{\omega}^2 = k_y^2 u^2 A^2/(A+B)^2$. The final expressions for the limiting parameters have the form

$$\omega^2 = k_y^2 u^2 k^4 \frac{m_n^2}{m_p^2}\frac{p_0^2}{n_0^2} \Big/ \left(k_z^2 + k^2\frac{m_n p_0}{m_p n_0}\right)^2; \tag{17}$$

$$k_y^2 u^2 = \nu\left(k_z^2 + k^2\frac{m_n p_0}{m_p n_0}\right)^2\left(k_z^2\nu + k^2\frac{m_n p_0}{m_p n_0}\nu_R\right) \Big/ \frac{\omega_{0p}^2}{c^2}\,k^4. \tag{18}$$

Taking the concentration gradient into account, these expressions become considerably more complicated; moreover, the threshold value of the velocity may increase substantially. It follows from (18) that the minimum value of the threshold velocity

$$u^2{}_{\min} \simeq \nu\nu\, c^2/\omega_{0p}^2 \tag{19}$$

is attained for $k_x \sim k_y \sim k;\ k_z/k \simeq m_n p_0/m_p n_0 \ll 1$.

Since ν falls with decreasing k, the smallest threshold corresponds to the excitation of the longest-wavelength oscillations. In this case the wavelength is determined by the smallest size of the system, $k_{\min} \sim \pi/L_{\min}$, if there is no mechanism permitting modes with $k < k_{\min}$ to develop (6). It is significant that under real conditions the threshold determined by formula (19) corresponds to velocities considerably smaller than the thermal velocities of the carriers.

The increment of growth of the oscillations near the threshold is found in the usual way—by expanding the dispersion relation near the threshold in the small quantities $\Delta\omega = \omega - \omega$, $\Delta u = u - u$

$$\gamma = 2\frac{\Delta u}{u}\nu\,. \tag{20}$$

We note that the expression found for the oscillation frequency at the stability boundary refers to the coordinate system in which the plasma as a whole is at rest. Therefore, in the laboratory coordinate system the experimentally observed frequency spectrum should be

$$0 < f \lesssim \frac{1}{2\pi}\left(\omega_{\mathrm{p}} + k_{x\,\min} v_{\mathrm{dr}}\right), \qquad v_{\mathrm{dr}} \cong c\,\frac{E_0}{H_0}. \tag{21}$$

Let us estimate the threshold velocity u_{p} and the oscillation frequency f for InSb at $T = 300^\circ$K, where $m_p/m_n \cong 30$, $\nu \cong 10^{12}$ sec^{-1}, $\nu_R \cong 10^7$ sec^{-1}, $n_0 \cong p_0 \cong 1.5 \cdot 10^{16}$ cm^{-3}. According to (19), (21), we obtain, for $H \gtrsim 3$ kOe, $u_{\mathrm{p}}\, 10^6$ cm/sec $\ll v_{Tp}$, $0 < f \lesssim 30$ MHz.

The authors express their gratitude to L. I. Rudakov for his constant interest in the work.

Received
10 XII 1968

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Note: Figure translations are in progress. See original paper for figures.