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Abstract

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MECHANICS

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ON THE DYNAMICS OF ONE MODEL OF THE RELATIVE MOTION OF A SHIP AND A COSMONAUT DURING APPROACH BY MEANS OF A CABLE

(Presented by Academician B. N. Petrov, 9 VIII 1968)

The model of the system considered in this work is shown in Fig. 1, where X_0OY_0 is an inertial coordinate system with origin at the center of the Earth; rigid body 1 is the ship with center of mass O_1; point mass 3 is the cosmonaut; O' is the center of mass of the cosmonaut-ship system. The cable is regarded as a nonstationary constraint varying according to the linear law $r_2 = r_{20} + \dot{r}_{20}t$. Assuming the angular motion of the system, described by the generalized coordinates φ_1 and α, to be unperturbed during hauling-in by external forces, we may write the kinetic potential of the system in the following form:

$$L = {}^1/_2\dot{q}'A\dot{q} + \dot{q}'B + T_0 - \Pi, \tag{1}$$

where

$$T_0 = {}^1/_2 k\dot{r}_{20}^2, \qquad \Pi = \Pi(\rho,\theta),$$

$$A = \begin{pmatrix} m_1+m_3 & 0 & 0 & 0 \\ 0 & (m_1+m_3)\rho^2 & 0 & 0 \\ 0 & 0 & J_1+kl_t^2 & -kr_2(b+r_2) \\ 0 & 0 & -kr_2(b+r_2) & kr_2^2 \end{pmatrix},$$

$$B = \begin{pmatrix} 0 \\ 0 \\ -ka\dot{r}_{20} \\ 0 \end{pmatrix}, \qquad \dot{q} = \begin{pmatrix} \dot{\rho} \\ \dot{\theta} \\ \dot{\varphi}_1 \\ \dot{\alpha} \end{pmatrix},$$

$$k = (m_1m_3)/(m_1+m_3), \qquad a = r_1\sin\alpha, \qquad b = r_1\cos\alpha, \qquad l_t^2 = r_1^2+r_2^2+2br_2.$$

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Here Π is the potential of the gravitational forces; J_1, m_1 are the moment of inertia and mass of the ship, and m_3 is the mass of the cosmonaut. The generalized coordinate φ_1 does not enter explicitly into the expression for the kinetic potential, i.e., it is cyclic, or ignorable, which makes it possible to reduce the order of the system by two units by using, for example, the equations of motion in the Routh form ([1]). It follows also from the structure of L that the motion of the center of mass and the angular motion of the system can be considered independently. For the Routh variables $\varphi_1, \alpha, p_1, \dot{\alpha}$, where

$$p_1 = \partial L/\partial\dot{\varphi}_1 = (J_1 + kl_t^2)\dot{\varphi}_1 - kr_2\dot{\alpha}(b + r_2) - ka\dot{r}_{20},$$

the equations of the angular motion of the system have the form

$$\ddot{\alpha} = \frac{1}{r_2}\left[-\frac{a(b + r_2)F_2}{J_1} - a\dot{\varphi}_1^2 + 2\dot{r}_{20}(\dot{\varphi}_1 - \dot{\alpha})\right], \tag{2}$$

$$d\varphi_1/dt = \partial R/\partial p_1, \qquad dp_1/dt = -\partial R/\partial\varphi_1,$$

where

$$F_2 = [kb\dot{\varphi}_1^2 + kr_2(\dot{\varphi}_1 - \dot{\alpha})^2]J_1/(J_1 + ka^2)$$

is the cable tension force, and R is the Routh function. Since R does not depend on φ_1, $p_1 = p_{10} = \text{const}$, and

$$d\varphi_1/dt = \partial R/\partial p_1 = (p_{10} + kr_2(b + r_2)\dot{\alpha} + ka\dot{r}_{20})/(J_1 + kl_t^2), \tag{3}$$

the investigation of the equations of motion of the system relative to the center of mass reduces to the investigation of a second-order equation:

$$\ddot{\alpha} = -\frac{J_1 + k(r_1^2 + br_2)}{J_1 + kl_t^2}\left(\frac{2\dot{r}_{20}}{r_2}\dot{\alpha} + \frac{ka(b + r_2)}{J_1 + ka^2}\dot{\alpha}^2\right) - \frac{(p_{10} + ka\dot{r}_{20})(p_{10}a - ka^2\dot{r}_{20} - 2r_{20}J_1)}{r_2(J_1 + ka^2)(J_1 + kl_t^2)}. \tag{4}$$

and two quadratures, one of which, $p_1 = p_{10}$, expresses the law of conservation of the kinetic moment of the system relative to its center of mass, while the other

$$\varphi_1 = \int_{t_0} \frac{\partial R}{\partial p_1}\,dt + \varphi_{10}$$

can be computed after integrating equation (4).

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Fig. 1. Diagram of the system and generalized coordinates

Figure 1: Fig. 1. Diagram of the system and generalized coordinates

Consider the equation of relative motion of the spacecraft and the cosmonaut (4). The positions of relative equilibrium of the system a_i^* are obtained from equation (4) for $\dot{a} = \ddot{a} = 0$. They are expressed in terms of the auxiliary quantities

$$a_i^* = r_1 \sin \alpha_i^*, \tag{5}$$

which take three values:

$$a_1^* = -\frac{p_{10}}{k\dot{r}_{20}}, \qquad a_{2,3}^* = \frac{p_{10}}{2k\dot{r}_{20}} \pm$$

$$\pm p_{10}\sqrt{\left(\frac{1}{2k\dot{r}_{20}}\right)^2 - \frac{2J_1}{kp_{10}^2}}, \tag{6}$$

to each of which, in the interval $(-\pi, \pi]$, there correspond two values of α_i^*: $\alpha_1^*, \alpha_1^{*'}, \alpha_2^*, \alpha_2^{*'}, \alpha_3^*, \alpha_3^{*'}$.

Fig. 1. Diagram of the system and generalized coordinates

For $\alpha^* = \alpha_1^*, \alpha^* = \alpha_1^{*'}$, the cyclic velocity $\dot{\varphi}_1 = 0$, and the motion of the system is an inertial approach of a nonrotating spacecraft and a cosmonaut with zero cable tension; whereas for $\alpha^* = \alpha_{2,3}^*, \alpha^* = \alpha_{2,3}^{*'}$, the motion of the system is a spin-up of the system in inertial space with velocity $\dot{\varphi}_1$, with the position of the cable relative to the spacecraft unchanged. The dependence of the positions of relative equilibrium on the parameters (the bifurcation diagram) is shown in Fig. 2, where I is the curve

$$p_{10} = -ka_1^*\dot{r}_{20},$$

and II and II' are the curves

$$p_{10} = \pm\left[ka_{2,3}^*\dot{r}_{20} + \frac{2\dot{r}_{20}J_1}{a_{2,3}^*}\right],$$

having the form II or II' depending on whether the quantity

$$a_m^* = \sqrt{2J_1/k}$$

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takes values smaller or larger than r_1. The values of the parameter p_{10}: p'_{10}, $\bar{p}_{10}$, and $p_{10}^{\min}$, at which the number of equilibrium positions changes, are bifurcation values and are determined by the formulas

$$p'_{10} = |\dot{r}_{20}| \left[\frac{2J_1}{r_1} + kr_1 \right], \qquad p_{10}^{\min} = 2\sqrt{2}\,|\dot{r}_{20}|\sqrt{kJ_1}, \qquad \bar{p}_{10} = kr_1|\dot{r}_{20}|.$$

In Fig. 2, for the bifurcation diagram of type II', the principal types of relative motion of the spacecraft and the cosmonaut are presented for $p_{10} > p'_{10}$ and $p_{10} < p'_{10}$. We investigate the behavior of the system in neighborhoods of the positions of relative equilibrium, linearizing equation (4). In neighborhoods of $\alpha_2^*, \alpha_2^{*'}, \alpha_3^*, \alpha_3^{*'}$, these equations have the form

$$\bar{\alpha}'' + \frac{2\left[J_1 + k\left(r_1^2 + b^* r_2\right)\right]}{r_2\left(J_1 + kl_t^{*2}\right)}\bar{\alpha}' + \frac{2b^*(2J_1 - ka^{*2})}{a^{*2} r_2\left(J_1 + kl_t^{*2}\right)}\bar{\alpha} = 0, \tag{7}$$

and in neighborhoods of α_1^* and $\alpha_1^{*'}$ they have the form

$$\bar{\alpha}'' + \frac{2\left[J_1 + k\left(r_1^2 + b^* r_2\right)\right]}{r_2\left(J_1 + kl_t^{*2}\right)}\bar{\alpha}' - \frac{2kb^*}{r_2\left(J_1 + kl_t^{*2}\right)}\bar{\alpha} = 0, \tag{8}$$

where $\bar{\alpha} = \alpha - \alpha^*$; an asterisk denotes the value of the corresponding quantities at $\alpha = \alpha^*$, and $\bar{\alpha}' = d\bar{\alpha}/dr_2$. Equations (7) and (8) are equations

of the Fuchs class [(2)] and have 4 singular points of pole type. The problem of integrating such equations has not yet been solved. In this work a qualitative investigation and asymptotic integration of equations (7) and (8) were carried out. Using known theorems [(3)], one can establish that the solutions of equations (7) and (8) in neighborhoods of $\alpha_2^{*}{}'$, α_3^*, and α_1^* cannot be oscillatory, while in neighborhoods of α_2^*, $\alpha_3^{*}{}'$, and $\alpha_1^{*}{}'$ they will be oscillatory on a sufficiently large interval of variation of the argument. With the aid of the Sonin–Pólya theorem [(3)], one can establish that the sequence of possible extrema of the oscillatory solutions of equations (7) and (8) is decreasing on the interval $[r_{20}, \bar{r}_2]$ of variation of r_2 and increasing on the interval $(\bar{r}_2, 0)$, where $\bar{r}_2 = \sqrt{(J_1 + kr_1^2)/k}$. Integrating equations (7) and (8) in a neighborhood of the singular point $r_2 = 0$ by the Frobenius method [(3)], one can establish that the zeros of the oscillatory solutions have no points of accumulation, while the derivative of the general solution tends to infinity. An asymptotic representation of the solution possessing all the listed properties can be constructed by using the fact of a "slow" variation of the coefficients of equations (7) and (8), introducing, for example, a large parameter [(4)]. For the equation

Fig. 2. Bifurcation diagram and main types of motion

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Figure 2

Figure 2: Figure 2

$$\bar{\alpha}'' + A(r_2)\bar{\alpha}' + B(r_2)\bar{\alpha} = 0,$$

which is the general form of equations (7) and (8), the asymptotic representation of the solution has the form

$$\bar{\alpha} = \exp\left[-\frac{1}{2}\int_{r_{20}}^{r_2} A(r_2)\,dr_2\right] I^{-1/4}(r_2)\left\{C_1 \exp\left[i\int_{r_{20}}^{r_2}\sqrt{I(r_2)}\,dr_2\right] + \right.$$

$$\left. +C_2 \exp\left[-i\int_{r_{20}}^{r_2}\sqrt{I(r_2)}\,dr_2\right]\right\}, \quad (9)$$

where

$$I(r_2) = B(r_2) - \frac{1}{4}A^2(r_2) - \frac{1}{2}\,dA/dr_2,$$

except, possibly, for a very small neighborhood of the point $r_2 = 0$.

The quantitative estimate of the approximation can be judged from Fig. 3, which shows the numerical solution of the nonlinear equation and the asymptotic solution of the linear equation (7), constructed for α_2^*. Using the method of analysis set forth and a digital computer, one can construct the domain of initial distances (r_{20})–tangential velocities ($v = r_{20}\dot{\alpha}_0$) of the cosmonaut-

where the inequalities are satisfied:

$$|\alpha(t)| \le \alpha_m, \qquad |\varphi_1(t)| \le \varphi_m, \qquad |F_2(t)| = F_2(t) \le F_m, \quad (10)$$

the satisfaction of which ensures the cosmonaut' s return.

These regions (one of which is shaded) are shown in Fig. 4 for $J_1 = 700$ kg m sec^2, $k = 20$ kg sec^2/m, $\alpha_0 = 0$, $\alpha_m = \pi/2$, for various fixed F_m and φ_m. In Fig. 4 the notation is as follows: I–curves between which the first of inequalities (10) is satisfied for $\varphi_{10} = 0$ and $\dot{\alpha} \ne 0$; II–the curve above which the first of inequalities (10) is satisfied for $\dot{\alpha}_0 = 0$ and $\varphi_{10} \ne 0$; φ_m, F_m –curves below which the second and third of inequalities (10) are satisfied. It is evident that the tangential velocities of the cosmonaut for any distance must be bounded above and below by the return conditions. Apparently, this circumstance excludes the possibility of using an uncontrolled cable system for

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Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

returning the cosmonaut to the spacecraft at a constant cable-winding speed, since the initial distances r_{20} and initial tangential velocities v_τ may take values that do not belong to the specified region.

Fig. 3. Asymptotic solution of the linear equation and solution of the nonlinear equation

Fig. 4. Regions of admissible initial distances–tangential velocities of the cosmonaut

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