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NATURAL VIBRATIONS AND FLUTTER OF THREE-LAYER CYLINDRICAL SHELLS IN A SUPERSONIC GAS FLOW

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1. The equations of elastic equilibrium of a three-layer cylindrical shell, when the assumptions of the semimembrane theory are used, can be represented in the form ([1])

$$D\frac{\partial^4}{\partial s^4}\left(\frac{\partial^2}{\partial s^2}+\frac{1}{R^2}\right)^2\left(1-\frac{\vartheta h^2}{\beta}\frac{\partial^2}{\partial s^2}\right)\chi+\frac{Eh}{R^2}\frac{\partial^4}{\partial x^4}\left(1-\frac{h^2}{\beta}\frac{\partial^2}{\partial s^2}\right)\chi=\frac{1}{R}\frac{\partial^2 p}{\partial s^2}, \tag{1}$$

where D, E, R, h are the cylindrical stiffness, Young's modulus, radius, and thickness of the shell; ϑ, β are parameters characterizing, respectively, the bending stiffness of the load-bearing layers and the shear stiffness of the core; $\chi(x, s, t)$ is a displacement function related to the deflection w by the relation

$$w=\left(1-\frac{h^2}{\beta}\frac{\partial^2}{\partial s^2}\right)\chi. \tag{2}$$

The function p is determined through the normal component q and the tangential components p_1, p_2 of the external surface load,

$$p = R\,\partial^2 q/\partial s^2 + \partial p_2/\partial s - \partial p_1/\partial x. \tag{3}$$

Let us assume that the shell is washed on the outside by a supersonic gas flow directed along the generator. Using the linear approximation of piston theory, we represent the normal load in the form

$$q = N_x\frac{\partial^2 w}{\partial x^2}+N_s\left(\frac{\partial^2 w}{\partial s^2}+\frac{w}{R^2}\right)-\Omega\frac{\partial^2 w}{\partial t^2}-\frac{\varkappa p_0}{a_0}\left(\frac{\partial w}{\partial t}+V\frac{\partial w}{\partial x}\right), \tag{4}$$

where N_x, N_s are line forces in the longitudinal and circumferential directions; Ω is the specific density of the three-layer package of shell material; p_0, a_0, V are the static pressure, speed of sound, and speed of the undisturbed flow; $\varkappa = c_p/c_v$.

The tangential forces will be ([1])

$$p_1 = 0, \qquad p_2 = N_x \frac{\partial^4}{\partial x^2 \partial s^2} \left(\frac{\partial^2 w}{\partial s^2} - \frac{w}{R^2} \right). \tag{5}$$

We represent the displacement function in the form

$$\chi(x, s, t) = \chi(x, s) e^{i\omega t}, \tag{6}$$

where ω is the complex vibration frequency.

Substituting (2)–(6) into (1) and passing to dimensionless parameters and coordinates $x/l,\ s/R$ (l is the length of the shell), while retaining the former notation for the coordinates, we obtain

$$\frac{\partial^4}{\partial s^4} \left(\frac{\partial^2}{\partial s^2} + 1 \right)^2 \left(1 - \vartheta k \frac{\partial^2}{\partial s^2} \right) \chi + \Theta_* \frac{\partial^4}{\partial x^4} \left(1 - k \frac{\partial^2}{\partial s^2} \right) \chi -$$

$$- \left[n_x \frac{\partial^4}{\partial x^2 \partial s^2} \left(\frac{\partial^2}{\partial s^2} - 1 \right) + n_s \frac{\partial^4}{\partial s^4} \left(\frac{\partial^2}{\partial s^2} + 1 \right) \right] \left(1 - k \frac{\partial^2}{\partial s^2} \right) \chi -$$

$$- \omega_*^2 \left(1 - k \frac{\partial^2}{\partial s^2} \right) \chi + \left(i\varepsilon\omega_* + p_* \frac{\partial}{\partial x} \right) \left(1 - k \frac{\partial^2}{\partial s^2} \right) \chi = 0. \tag{7}$$

Here

$$k = \frac{9h^2}{\beta R^2}, \qquad \Theta_* = \frac{12}{\Theta} \left(\frac{R}{n} \right)^2 \left(\frac{R}{l} \right)^4, \qquad n_x = \frac{N_x R^2}{D} \left(\frac{R}{l} \right)^2, \qquad n_s = \frac{N_s R^2}{D},$$

$$\omega_*^2 = \frac{\Omega R^4}{D} \omega^2, \qquad p_* = \frac{\chi p_0 R^3}{D} \frac{R}{l} \mathrm{M}, \qquad \varepsilon = \frac{\chi p_0 R^2}{\sqrt{\Omega D}} \tag{8}$$

(Θ is the dimensionless parameter (1), $\mathrm{M} = V/a_0$ is the Mach number of the undisturbed flow).

We represent the solution of (7) in the form

$$\chi(x, s) = \sin(ns) \chi(x), \tag{9}$$

where n is the prescribed number of waves in the circumferential direction. Substituting (9) into (7), we find

$$\Theta_* \frac{d^4\chi}{dx^4} - n_x(1+n^2)n^2 \frac{d^2\chi}{dx^2} + p_* n^4 \frac{d\chi}{dx} + \left[\frac{1+\vartheta k n^2}{1+kn^2}(1-n^2)n^4 - n_s(1-n^2)n^4 - \omega_*^2 n^4 + i\varepsilon\omega_* n^4\right]\chi = 0. \tag{10}$$

A particular solution of equation (10) will be

$$\chi(x) = e^{i\alpha x}. \tag{11}$$

Substituting (11) into (10), we obtain the algebraic equation

$$a_4\alpha^4 + a_2\alpha^2 + a_1\alpha + a_0 = 0, \tag{12}$$

where

$$a_4 = \Theta_*, \qquad a_2 = n_x(1+n^2)n^2, \qquad a_1 = in^4 p_*,$$

$$a_0 = \frac{1+\vartheta k n^2}{1+kn^2}(n^2-1)^2 n^4 - n_s(n^2-1)n^4 - \omega_*^2 n^4 + i\varepsilon\omega_* n^4. \tag{13}$$

Determining the roots α_i of equation (12), we construct the general solution (for a given n) of (10)

$$\chi(x) = \sum_{i=1}^{4} c_i e^{i\alpha_i x}. \tag{14}$$

2. Let us consider several boundary-value problems.

A. Hingedly supported edges. In this case, for the function one can formulate two conditions at each edge ($x = 0,\ x = 1$) of the form

$$\chi = \chi'' = 0. \tag{15}$$

Substituting (14) into (15), we arrive at a system of linear algebraic equations with respect to the coefficients c_i. The condition for a nontrivial solution will be

$$\Delta\delta^{-1} = 0, \tag{16}$$

where $\Delta(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ is the determinant of the indicated system, and δ is the Vandermonde determinant formed from the roots α_i. The determinant Δ is a sum of 6 terms of the form

$$(\alpha_1^2 - \alpha_2^2)(\alpha_3^2 - \alpha_4^2)e^{i(\alpha_3+\alpha_4)}. \tag{17}$$

The remaining terms are obtained from (17) by cyclic permutation of the indices $1, 2, 3, 4$.

B. Clamped edges. The boundary conditions will be

$$\chi = \chi' = 0. \tag{18}$$

The determinant Δ is composed of terms of the form

$$(\alpha_1 - \alpha_2)(\alpha_3 - \alpha_4)e^{i(\alpha_3+\alpha_4)}. \tag{19}$$

Fig. 1 Fig. 2

Fig. 1. Effect of aerodynamic damping on the critical flutter speed of a three-layer cylindrical shell with simply supported edges; $\vartheta = 0.05$; $k = 1.00$; $n = 12$

Fig. 2. Variation of the real part of the dimensionless vibration frequency ω_*' of a three-layer cylindrical shell in a supersonic flow (simply supported edges); $\vartheta = 0.05$; $k = 1.00$; $n = 12$

Fig. 3 Fig. 4

Fig. 3. Effect of the shear stiffness of the core and the bending stiffness of the face layers on the critical flutter speed in a supersonic flow; the shell edges are simply supported; $\Theta_* = 100$; $\varepsilon = 0.1$; $n = 12$

Fig. 4. Effect of compressive forces in the circumferential direction on the vibration frequency of a three-layer shell in a supersonic flow (simply supported edges, frequencies 1–2); $\vartheta = 0.05$; $k = 1.00$; $n = 12$

B. For a cantilever-clamped shell we have

$$\chi = \chi' = 0 \quad (x = 0); \qquad \chi'' = \chi''' = 0 \quad (x = 1), \tag{20}$$

and the first term of the determinant Δ is equal to

$$(a_1 - a_2)a_3^2a_4^2(a_3 - a_4)e^{i(a_3+a_4)}. \tag{21}$$

For a clamped and simply supported shell, respectively, we obtain

$$\chi = \chi' = 0 \quad (x = 0); \qquad \chi = \chi'' = 0 \quad (x = 1); \tag{22}$$

$$(a_1 - a_2) a_3^2 a_4^2 (a_3 - a_4) e^{i(a_3 + a_4)}. \tag{23}$$

Systematic numerical calculations were carried out on the "Strela" electronic digital computer over a wide range of dimensionless parameters. Some of the results are shown in Figs. 1–4. The calculations make it possible to clarify a number of interesting, previously unknown features of the behavior of a three-layer shell in a gas flow. The principal ones are as follows. The role of aerodynamic damping in determining the critical flutter speed depends to a considerable degree on the geometry of the shell itself (the parameter Θ_*, Fig. 1). Neglecting aerodynamic damping may lead to substantial errors in determining the flutter speed (Figs. 1 and 2). Other conditions being equal, the flutter speed of a three-layer shell decreases rapidly as the shear stiffness of the filler decreases (as the coefficient k increases, Fig. 3). Compression of the shell in the longitudinal and circumferential directions contributes to a reduction in the flutter speed (Fig. 4). The number of waves in the circumferential direction corresponding to the minimum flutter speed varies, depending on the boundary conditions, in the range $n = 12$–18. At the same time, more rigid boundary conditions correspond to a larger value of n.

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1. E. I. Grigolyuk, P. P. Chulkov, *Critical Loads of Three-Layer Cylindrical and Conical Shells*, Novosibirsk, 1966.

Note: Figure translations are in progress. See original paper for figures.