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Abstract

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HEAT TRANSPORT IN A PLASMA DUE TO AN INSTABILITY ON TRAPPED ELECTRONS

In work ([1]) it was shown that in a comparatively dense high-temperature plasma confined in a toroidal magnetic field, excitation of drift waves on trapped electrons is possible in a certain range of parameters. Estimates were also made there of the effective diffusion and thermal-conductivity coefficients for a developed instability for the case of systems with a large shear $\theta \gg \rho/a$, where ρ is the ion Larmor radius at the electron temperature; a is the minor radius of the plasma torus; θ is the shear, which for axisymmetric systems of the Tokamak type is defined as

$$\theta = \frac{r^2}{Rq^2}\frac{dq}{dr};$$

r is the distance from the magnetic axis; R is its radius of curvature, $q = rB_z/RB_\vartheta$ is the safety factor with respect to the kink instability. The estimates of the transport coefficients were based on the assumption that it is precisely the shear that limits the amplitude of the oscillations and that convective cells with localization width $\Delta \lesssim \rho/\theta \ll a$ develop in the plasma. If θ is small, then another mechanism for limiting the amplitude of the oscillations comes into play. Namely, when the amplitude of the oscillations of the electric-field potential φ approaches $\frac{r}{R}\frac{T}{e}$, the effect of electric trapping of particles in the drift wave becomes substantial ([2]). Allowance for this effect and determination of the corresponding electron thermal-conductivity coefficient at small θ constitute the subject of the present article.

For definiteness, let us consider an axisymmetric system of the Tokamak type with $B_z \gg B_\vartheta$. Introduce the coordinate z along the field lines, measured from the outer circumference. Then, neglecting magnetic drift in a weakly inhomogeneous magnetic field, the kinetic equation for electrons in the drift approximation can be written in the form

$$\frac{\partial f}{\partial t} + v_z\frac{\partial f}{\partial z} - \frac{1}{B}\frac{\partial B}{\partial z}\frac{v_\perp^2}{2}\frac{\partial f}{\partial v_z} + \frac{c[\mathbf{h}\nabla\varphi]}{B}\nabla f + \frac{e}{m}\frac{\partial \varphi}{\partial z}\frac{\partial f}{\partial v_z} = \mathrm{St}(f), \qquad (1)$$

where the third term on the left takes into account the effect of reflection of particles with small v_z from magnetic mirrors; $\mathrm{St}(f)$ is the collision term; $\mathbf{h} = \mathbf{B}/B,\ B(z) = B_0(1 - \varepsilon \cos k_0 z);\ \varepsilon = r/R;\ k_0 = 1/qR;\ \varphi$ is the electric-field potential.

In the linear approximation, equation (1), together with the quasineutrality condition $n'_e = n'_i$, leads to the conclusion about excitation of drift waves on trapped electrons ([1]). The corresponding perturbations are localized near the outer circumference of the torus, so that for $m = k_y r \gg 1$ they may be regarded as waves standing along the magnetic field and traveling in the azimuthal direction. Their amplitude on the inner circumference is zero. As the amplitude of the oscillations grows, electric trapping of particles becomes substantial; for $d \ln T / d \ln n > 0$, for waves standing in z, this plays a stabilizing role ([2]). At the same time, owing to "exchange" by trapped particles along $\mathbf{B}$, drift waves can increase the wavelength along the field lines and acquire an admixture of a wave traveling in z. But in order not to complicate the discussion, we shall assume the presence of a small shear θ, which should prevent the development of drift waves traveling in z with broad locali-

tion in r, and we shall assume that waves of finite amplitude are still localized on the outer contour. Their potential φ can be approximated by the expression

$$\varphi = \varphi_0 \cos(\omega t - k_y y)\,[{}^1/_2 + {}^1/_2 \cos k_0 z], \tag{2}$$

where y is the coordinate in the azimuthal direction, $k_0 = 1/qR$.

The amplitude φ_0 may be regarded as small compared with T_e/e, so that for the perturbation of the ion density n'_i one may use the linear approximation

$$n'_i = \frac{\omega_*}{\omega} \frac{e n_0 \varphi}{T_e}, \tag{3}$$

where $\omega_* = -k_y \frac{cT_e}{eBn_0} \frac{dn_0}{dr}$ is the drift frequency.

The perturbation of the electron density n'_e can be represented in the form $n'_e = n_0 \frac{e\varphi}{T_e} + n'_t$, where the first term corresponds to the main part of the electrons, distributed according to Boltzmann, and n'_t takes into account the deviation from the Boltzmann distribution of the trapped electrons. It is precisely n'_t that determines the magnitude of the increment in the linear approximation and the amplitude of the established oscillations.

To determine n'_t, we shall use equation (1) for trapped electrons. In doing so, instead of v_z it is convenient to introduce a new variable

$$E = \frac{mev_z^2}{2} - \varepsilon \frac{mev_\perp^2}{2} \cos x_0 z - e\varphi$$

and write (1) in the form

$$\frac{\partial f}{\partial t} + \frac{\partial E}{\partial t}\frac{\partial f}{\partial E} + v_z(E)\frac{\partial f}{\partial z} + \frac{c[\bar{h}\nabla\varphi]}{B}\nabla_\perp f = \mathrm{St}(f). \tag{4}$$

Since the longitudinal velocity of the electrons v_z is very large, equation (4) can be solved by the method of successive approximations. In the zeroth approximation we have $v_z(E)\partial f_0/\partial z = 0$, i.e. $f = f_0(E, r, v_\perp^2, t)$. From the solvability condition for the equation of the next approximation we find

$$\frac{\partial f_0}{\partial t} + \left\langle \frac{\partial E}{\partial t} \right\rangle \frac{\partial f_0}{\partial E} + \left\langle c\,\frac{[\bar{h}\nabla\varphi]}{B} \right\rangle \nabla_\perp f_0 = \langle \mathrm{St}(f_0) \rangle, \tag{5}$$

where the averaging is understood in the sense

$$\langle F \rangle = \oint F\,dz/v_z \Big/ \oint dz/v_z, \tag{6}$$

and the integrals for trapped particles are taken between the turning points.

For small wave amplitude the third term on the left-hand side of (5) may be taken in the linear approximation; it is sufficient to take the nonlinearity into account only in the trajectories of the trapped particles, i.e. in the expression $v_z(E)$. Taking into account that $\partial E/\partial t = -e\,\partial\varphi/\partial t$, $[\bar{h}\nabla\varphi]_r = -\partial\varphi/\partial y$, we write equation (5) for the distribution function of trapped electrons in the wave of the form (2) as

$$\frac{\partial f_0}{\partial t} + \left(e\omega\frac{\partial f_0}{\partial E} - \frac{c x_y}{B}\frac{\partial f_0}{\partial r} \right) A\varphi_0 \sin(\omega t - k_y y) = \langle \mathrm{St}(f) \rangle, \tag{7}$$

where $A = \langle (1 + \cos k_0 z)/2 \rangle$. Since the main contribution to n_t' is made by trapped electrons with small v_z, concentrated near the outer contour, one may approximately put $A = 1$.

In the second term on the left-hand side of (7), which is small $\sim \varphi_0$, small perturbations in the function f_0 may be neglected, i.e. it may be considered Maxwellian. Furthermore, since the first term of (7) has the order of magnitude ωf_0, and the instability develops for $\nu_{\mathrm{eff}} > \omega$ ([1]), then for growing perturbations the first term may be neglected in comparison with the collision term. As for the collision term, for the deviation from the Maxwellian distribution of the trapped particles f_t' we za-

we write it in a simplified form, introducing a certain effective collision frequency: $\langle \mathrm{St}(f) \rangle = -\nu_{\mathrm{eff}} f_t'$. Thus, from (7) we obtain

$$n_t' = \int f_t' d\bar{v} = \frac{e\varphi_0}{T_e}\sin(\omega t - k_y y)\int \frac{\omega - \omega_{*e}}{\nu_{\mathrm{eff}}} f_0 d\bar{v}, \tag{8}$$

where

$$\omega_{*e} = -\frac{ck_y T_e}{eBf_0}\frac{\partial f_0}{\partial r} = \omega_* \left[1 + \left(\frac{mv^2}{2T_e} - \frac{3}{2}\right)\frac{d\ln T_e}{d\ln n}\right].$$

Expression (8) for n'_t should be substituted into the quasineutrality condition $n'_i = n'_e = n_0 e\varphi/T_e + n'_t$. Here, according to (3), n'_i is in phase with $\varphi \sim \cos(\omega t - k_y y)$. Therefore, in a drift wave stationary in amplitude, the condition $\int n'_t \sin(\omega t - k_y y)\, dy = 0$ must be satisfied. It is precisely this condition that determines the amplitude of the stationary drift wave.

In this case the oscillation frequency is $\omega = \omega_*$, which follows from the quasineutrality condition averaged over y with weight $\cos(\omega t - k_y y)$.

The effective frequency ν_{eff} in (8) depends, generally speaking, on the phase of the wave $\omega t - k_y y$. For $\varphi > 0$, along with magnetic trapping there is electric trapping, so that, taking into account the differential character of the Coulomb collision term, one may approximately put

$$\frac{1}{\nu_{\text{eff}}} \simeq \frac{1}{\nu_e}\left(\frac{e\varphi}{T_e} + \varepsilon\frac{v^3}{v_e^3}\right). \tag{9}$$

It is not difficult to see that for $e\varphi/T_e \gtrsim \varepsilon$ the sign of n'_t changes, i.e., a stabilization effect takes place.

For $\varphi < 0$, trapping begins only at sufficiently large $v_\perp^2 \gtrsim e\varphi/m_e$, when reflection from the magnetic mirrors is more effective than expulsion by the electric field. Accordingly, the contribution from this region decreases, and at $\varphi \approx \varepsilon T_e/e$ stabilization of the drift wave sets in. Such a wave does not lead to particle transport (diffusion), but it can transport heat on trapped electrons. The heat flux

$$q_T = \int \left\langle f'_t \frac{c}{B}\frac{\partial\varphi}{\partial y}\right\rangle \frac{m_e v^2}{2}\, d\bar{v}.$$

Substituting here the expression for f'_t and performing the averaging, taking into account $\omega_* \ll \nu_e/\varepsilon$, we obtain, in order of magnitude,

$$q_T \approx -\varepsilon^{3/2}\nu_e a^2 n_0 \frac{dT_e}{dr}, \tag{10}$$

where a is the minor radius of the toroidal column.

This result has a simple physical meaning, namely, the corresponding coefficient of thermal conductivity χ can be represented in the form $\chi = \sqrt{\varepsilon}\,\frac{\nu_e}{\varepsilon}(a\varepsilon)^2$, where the first factor takes into account the fraction of trapped particles, $\nu_e/\varepsilon = \nu_{\text{eff}}$, and the characteristic length εa corresponds to the radial displacement of a particle in a drift wave with amplitude $\varphi \sim \varepsilon T/e$.

The laminar convection considered here is possible only at sufficiently small ν_e, when $k_y < 1/\varepsilon a$, i.e. $\nu_e < \nu_1 \equiv cT_e/eBa^2$. For larger ν_e, and correspondingly $k_y > 1/\varepsilon a$, the nonlinear effect of limiting the oscillation amplitude at a level corresponding to a density gradient in the wave of the order of the main one is more substantial, i.e. $k_y\varphi \sim T_e/ea$. The corresponding coefficient of turbulent thermal conductivity, estimated by us in (1), has order of magnitude

$$\chi \sim \varepsilon^{3/2}\frac{1}{\nu_e}\left(\frac{cT_e}{eBa}\right)^2.$$

Thus, over the entire range of variation of ν_e, the coefficient of electron thermal conductivity on trapped electrons in systems with small shear can be approximated by the expression

$$\chi \approx \varepsilon^{3/2}a^2\frac{\nu_e}{1+\nu_e^2/\nu_1^2}, \tag{11}$$

where $\nu_1 = cT_e/eBa^2$.

As for diffusion, in the case of large collisions $\nu_e > \nu_1$, when turbulent convection develops, one should expect a diffusion coefficient D comparable to the coefficient of thermal conductivity χ. With laminar convection ($\nu_e < \nu_1$), in the approximation considered here $D = 0$; however, if in the expression for the perturbed ion density one takes into account terms of the order of the magnetic drift, then the frequency ω will be shifted relative to the drift frequency ω_* by an amount of order $\varepsilon \sim a/R$, which will lead to a particle flux with diffusion coefficient $D \sim \dfrac{a}{R}\chi$.

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REFERENCES

[1] B. B. Kadomtsev, O. P. Pogutse, DAN, 186, No. 3 (1969).
[2] B. B. Kadomtsev, O. P. Pogutse, DAN, 188, No. 1 (1969).

Note: Figure translations are in progress. See original paper for figures.