On the Angular Distribution in Elastic Scattering of High-Energy Hadrons

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On the Angular Distribution in Elastic Scattering of High-Energy Hadrons

It is of great interest to sum a series for the scattering amplitude under fairly general assumptions about the behavior of the partial amplitudes. Such an attempt in the case of backward scattering was made by D. I. Blokhintsev (^{1,2}); however, the calculations were of an orientational character and did not possess a sufficient degree of accuracy. Subsequently these calculations were revised and somewhat refined (^{3,4}). In work (^{4}) we considered not only the case of scattering at 180°, but also at other angles. In particular, in the case of scattering at 180° it was shown that it is determined by derivatives (differences) of the partial amplitudes. Recently, in connection with the appearance of sufficiently accurate experimental data on high-energy hadron scattering at large angles and in connection with the growing theoretical interest in this circle of questions, we have again returned to the question of summing the series for the scattering amplitude (^{5,6}).

In articles (^{5,6}), within the framework of the simplest model (^{5}), corresponding to the existence in the scatterer of a certain core on which scattering at large angles takes place, the question of the ratio of the real part of the scattering amplitude to its imaginary part in the region of the diffraction peak was considered. In (^{5}) it was indicated that the proposed model qualitatively describes well the differential cross section for high-energy hadron scattering over the entire angular region $0 \leq \theta \leq 180°$. In the present communication we shall show, using the examples of π^+p-scattering at momentum $p_l = 4$ GeV/c, π^-p-scattering at $p_l = 8$ GeV/c, and pp-scattering at $p_l = 21$ GeV/c, that this is indeed so.

The imaginary and real parts of the elastic-scattering amplitude for angles $0 \leq \theta < \pi - \varepsilon$, where $\varepsilon > \gamma > 0$, γ determining the region of the backward peak, in the model adopted are given by the formulas (^{6})

$$\operatorname{Im} f(\theta) \simeq \frac{k\sigma_t}{4\pi} e^{\frac{a}{2}t}, \qquad a = \sigma_t^2/16\pi\sigma_{el}, \tag{1}$$

$$\operatorname{Re} f(\theta) \simeq \left(\frac{\theta}{\sin\theta}\right)^{1/2} 6\delta\,\frac{k\sigma_t}{4\pi}\left(F(k\rho_0\theta) - \frac{J_0(k\rho_0\theta)}{12(k\rho_0)^2}\right), \tag{2}$$

where $-t = 2k^2(1-\cos\theta)$ is the invariant square of the transferred four-momentum; k is the wave number in the center-of-inertia system of the colliding particles; σ_t is the total interaction cross section; σ_{el} is the total elastic-scattering cross section due to the imaginary part of the amplitude; δ is the ratio of the real part of the forward scattering amplitude to its imaginary part; ρ_0 is a parameter characterizing the core, and the function $F(x)$ is determined by the formula

$$F(x) \simeq \begin{cases} \frac{4}{3}x^{-2}J_2(x), & x \leqslant 1, \\ \sum\limits_{n=2}^{\infty} \frac{(2n-5)!!}{x^n} J_n(x), & x \geqslant 1, \quad (-1)!! \equiv 1. \end{cases} \tag{3}$$

The scattering amplitude in the region of the backward peak ($\gamma \leq \theta \leq 180°$), as calculations have shown, is real in our model and is determined by the formula

$$f(\theta) \simeq f(180°)\left[1 - A_1\frac{(k\rho_0)^2}{2}(1+\cos\theta) + A_2\frac{(k\rho_0)^4}{16}(1+\cos\theta)^2 - \right.$$
$$\left. - A_3\frac{(k\rho_0)^6}{288}(1+\cos\theta)^3\right], \tag{4}$$

where

$$\begin{aligned} A_1 &\simeq 1 - 2(k\rho_0)^{-2}, \\ A_2 &\simeq 1 - 7.5(k\rho_0)^{-2} + 10(k\rho_0)^{-4}, \\ A_3 &\simeq 1 - 20(k\rho_0)^{-2} + 100(k\rho_0)^{-4}. \end{aligned} \tag{5}$$

For $x \gg 1$, for the function $F(x)$ we have (6)

$$F(x) \simeq \sum_{n=2}^{n_0-1} \frac{(2n-5)!!}{x^n} J_n(x) + \frac{1}{x^2\sqrt{\pi}}\left(\frac{\sqrt{x}}{x}\Phi\left(\frac{x}{2\sqrt{n_0}}\right) - \frac{1}{\sqrt{n_0}}\exp\left(-\frac{x^3}{4n_0}\right)\right), \tag{6}$$

where $n_0 \gg 1,\ x/n_0 < 1/2,\ \Phi(x)$ is the probability integral.

Figures 1-3 show the differential cross sections for elastic π^+p-scattering at momentum in the L-system $p_L = 4$ GeV/c, π^-p-scattering at $p_L = 8$ GeV/c, and pp-scattering at $p_L = 21$ GeV/c, respectively. The experimental data are taken from Refs. (7-9). The theoretical curves were constructed using formulas (1), (2), and (4); moreover, for $k\rho_0\theta \geq 1$ the calculations were carried out at integer values of $k\rho_0\theta$, and the points obtained in this way were connected by a

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

smooth curve. In the calculation the following numerical values were used for ρ_0 and δ: in the case of π^+p-scattering, $\rho_0 = 0.6$ fm, $|\delta| = 0.43$,

Fig. 1

Fig. 2

Fig. 3

in the case of π^-p-scattering, $\rho_0 = 0.7$ fm, $|\delta| = 0.12$, and in the case of pp-scattering, $\rho_0 = 1$ fm, $|\delta| = 0.15$. It should be noted that formula (2) is very sensitive to the value of the parameter ρ_0. In the region of small scattering angles the main contribution is made by the imaginary part of the amplitude, while in the region of large angles it is made by the real part. The contributions of the real and imaginary parts of the amplitude

of scattering become of the same order for momentum transfers (^{6})

$$t_0 \simeq 2a^{-1} \ln |\delta|. \tag{7}$$

If here one neglects the weak energy dependence of the logarithmic factor, then, as has already been shown (^{6}), result (7) is in qualitative agreement with the theory of complex angular momenta, in which a single vacuum pole is taken into account. If one assumes that δ varies asymptotically as a power, $\delta \sim s^{-\alpha}$, where $\alpha > 0$, then the agreement of result (7) with the theory of complex momenta becomes still better, if in the latter one takes into account (^{10}) the contribution of the Mandelstam branch points generated by the Pomeranchuk pole. A characteristic feature of the angular distribution predicted by our model is a change of regime at the square of the transferred momentum $-t$ in the region of 3 $(\mathrm{GeV}/c)^2$ in the case of π^+p- and π^-p-scattering and in the region $-t \simeq 1$ $(\mathrm{GeV}/c)^2$ in the case of pp-scattering; in addition, the differential cross section has an oscillatory character, theoretically first indicated by A. A. Anselm and I. T. Dyatlov (^{10}). From the theoretical and experimental results presented in Figs. 1–3, one may conclude that the proposed simplest model (^{5}) does indeed give a qualitatively rather good description of the differential cross section for elastic scattering of hadrons at high energies.

Fig. 3

Figure 3: Fig. 3

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Note: Figure translations are in progress. See original paper for figures.