GENERAL WAVEGUIDE THEORY AND THE LIMITING RESOLVING POWER OF MULTIBEAM SPECTRUM ANALYZERS

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GENERAL WAVEGUIDE THEORY AND THE LIMITING RESOLVING POWER OF MULTIBEAM SPECTRUM ANALYZERS

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With the development of coherent optics, interest has increased substantially in the classical problem of spectrum analysis. The need to measure the extremely fine spectrum of lasers poses the task of improving existing analyzers and, above all, increasing their resolving power. There is no doubt that, in addition, searches will be conducted for new methods of spectrum analysis. In this connection, the problem arises of determining the limiting capabilities of spectrum analyzers.

The purpose of the present work is to construct a general (waveguide) theory and to calculate the limiting resolving power of a broad class of spectrum analyzers—multibeam interferometers, to which the prism, diffraction grating, transmission and reflection Michelson echelons, the Lummer-Gehrcke plate, and the Fabry-Perot etalon may be assigned. In developing this theory, the theories of microwave frequency-scanning antennas ([1,2]) and equivalent waveguide circuits of multibeam interferometers are taken as the basis.

All multibeam interferometers are constructed according to one of two variants (series or parallel) of the equivalent circuits shown in Fig. 1. The discrete circuits depicted, in the case of an infinite number of emitters ($N \to \infty,\ T \to \infty$), also describe continuous circuits of analyzers. The difference in the operation of the series (Fig. 1a) and parallel (Fig. 1b) circuits reduces only to certain features of the amplitude distribution of the field over the radiating aperture. In the most general case, owing to radiation losses (series circuit) and losses in the waveguides (series and parallel circuits), an exponential distribution of amplitudes over the aperture, $e^{-\delta(z+L/2)}$, takes place.

Fig. 1. Generalized circuits of multibeam spectrum analyzers:
a—series; *b*—parallel. *A*—elementary emitters, *B*—waveguides, *D*—power dividers, *N*—number of emitters.

It is not difficult to show that, for the systems of Fig. 1, the directions of the principal maxima (beams) of the radiation pattern are described by the formula

Fig. 1. Generalized circuits of multibeam spectrum analyzers: a—series; b—parallel. A—elementary emitters, B—waveguides, D—power dividers, N—number of emitters

Figure 1: Fig. 1. Generalized circuits of multibeam spectrum analyzers: a—series; b—parallel. A—elementary emitters, B—waveguides, D—power dividers, N—number of emitters

$$\sin \varphi_m^k = k\lambda/T + \gamma_m l/T = k\lambda/T + l'\gamma_m, \tag{1}$$

where φ is the radiation angle, measured from the normal to the line of arrange-

of the emitters; λ is the wavelength; T is the distance between adjacent emitters; $\gamma_m = c/v_\phi$ is the slowing of the phase velocity v_ϕ in the waveguide of the m-th type of wave; c is the velocity of light in free space; l is the length of the waveguide sections between adjacent emitters; $l' = l/T$ is the relative length of the waveguides. It follows from (1) that, in the general case, the directions of the principal radiation maxima (beams) form a complicated multi-index set, where m is the index of the type of wave in the waveguides (in the general case, a double index), and k is the diffraction order of the spectrum.

By the angular-frequency sensitivity θ of the analyzer we shall mean the relative derivative of the radiation angle with respect to frequency, $d\varphi/(df/f)$ (or $-d\varphi/(d\lambda/\lambda)$). Differentiating (1) with respect to λ and eliminating the quantity k/T, we obtain:

$$\theta = \frac{1}{\cos\varphi}\left[l'\left(\gamma - d\gamma \Big/ \frac{d\lambda}{\lambda}\right) - \sin\varphi\right] = \frac{1}{\cos\varphi}(l'\gamma_{\mathrm{gr}} - \sin\varphi), \tag{2}$$

where $d\gamma/(d\lambda/\lambda)$ is the dispersion of the phase velocity in the waveguides feeding the emitters; $\gamma_{\mathrm{gr}} = \gamma - d\gamma/(d\lambda/\lambda) = c/v_{\mathrm{gr}}$ is the slowing of the group velocity v_{gr} in these waveguides (3).

In accordance with the definition of angular-frequency sensitivity, for a relative change in frequency df/f the beam is turned through an angle $d\varphi = \theta\, df/f$.

If by $\Delta f/f$ we denote the relative change in frequency for which the beam is displaced through a small angle β, equal to the beamwidth at half power, then $\beta = \theta\,\Delta f/f$. It is assumed here that, within the limits of a small change in angle, the angular-frequency sensitivity remains constant. Consequently, the resolving power of the analyzer is written in the form $R = f/\Delta f = \theta/\beta$. It can be shown that the width of the principal radiation maximum is $\beta = \dfrac{\lambda}{L\cos\varphi}\,a$, where a is the factor of beam broadening due to the deviation of the amplitude distribution from a uniform one and of the phase distribution from a linear one. Thus, taking (2) into account, we have

$$R = \frac{L}{a\lambda}(l'\gamma_{\text{gr}} - \sin\varphi). \tag{3}$$

Here $L/a\lambda$ is the aperture factor, characterizing the influence of the dimensions of the radiating system on the resolving power; $(l'\gamma_{\text{gr}} - \sin\varphi)$ is the dispersion factor, determined, for $\varphi = \text{const}$, by the parameters of the waveguides in the system.

The obtained expression for the resolving power is general for any spectrum analyzer of the type of multiple-beam interferometers and makes it possible to draw the following conclusions:

1. An increase in resolving power can be achieved not only by increasing the aperture factor, i.e., the length of the radiating system L, but also by increasing the dispersion factor, i.e., the group slowing γ_{gr} and the relative length of the waveguides $l' = l/T$.
2. The resolving power is maximal for the direction $\varphi = -90^\circ$ and minimal for the direction $\varphi = 90^\circ$. However, this dependence on angle is significant only for small γ_{gr}.
3. With nonuniform amplitude and nonlinear phase distributions of the field over the aperture, there is a certain decrease in resolving power due to the beam-broadening factor a.

Application of the general formula (3) to the various analyzers listed above leads to the known particular expressions for their resolving powers.

On the basis of the general relations obtained, let us consider the question of the limiting resolving power of multiple-beam interferometers. As

As is evident from (3), for a given value of L/λ, a substantial increase in resolving power is possible only by increasing the relative length of the waveguides l' or the group retardation γ_{gr} (by using waveguide modes close to critical ones, or by filling the waveguides with a retarding medium or structure). We shall show that in this case we arrive at a limit determined by the losses in the instrument. We rewrite relation (3) in the form:

$$R = \frac{Q}{l'\gamma_{\text{gr}}}\left(l'\gamma_{\text{gr}} - \sin\varphi\right). \tag{4}$$

Here $Q = 2\pi f W/P_{\text{loss}}$ is the quality factor of the waveguides used, where f is the frequency, W is the energy stored in a waveguide section of unit length, and P_{loss} is the loss power per unit length of the waveguide. Since, by assumption, $l'\gamma_{\text{gr}} \to \infty$, then

$$R(l'\gamma_{\text{gr}} \to \infty) = Q. \tag{5}$$

Thus, the maximum attainable resolving power of any spectrum analyzer of the multibeam-interferometer type, for a fixed length, is equal to the quality factor of the waveguides used. For waveguides not bounded by lossy walls (a prism, a diffraction grating, a Michelson echelon, a Lummer–Gehrcke plate in a regime close to total internal reflection), the waveguide quality factor coincides with the quality factor of the medium filling the waveguide, i.e., is equal to the cotangent of the loss angle of the medium. For waveguides bounded by absorbing walls (a Fabry–Perot interferometer), the quality factor is determined by the ratio of the waveguide volume to the area of the absorbing walls, by losses in the walls, and (to a small degree) by the field structure in the waveguide (the wave type).

Investigation shows that the spectrum analyzer in which the limiting resolving power—equal to the quality factor of the waveguide used and corresponding to $l'\gamma_{\text{gr}} \to \infty$—is realized is the Fabry–Perot interferometer (at small angles φ).

When the length L of the analyzer is increased, the beam width decreases and, according to (3), the resolving power increases. However, when L is so large that exponential attenuation of the wave along the aperture begins to have an effect, beam narrowing and the increase in resolving power cease. The resolving power in this case will evidently be determined by the same relation (4) as for fixed length. Thus, with an unlimited increase in the length of the radiating system of the spectrum analyzer, its resolving power likewise cannot exceed the quality factor of the waveguides used, i.e.,

$$R(L \to \infty) \leq Q. \tag{6}$$

It should be noted, however, that when waveguides of high quality factor are used—for example, free space as a waveguide for TEM waves (a diffraction grating, a reflective Michelson echelon)—from the standpoint of waveguide losses, an analyzer of practically unlimited length is possible. In these cases the resolving power is determined by the acceptable length of the instrument, which is limited by the attainable accuracy of its manufacture.

It follows from the relations obtained that a further increase in resolving power can be achieved by increasing the quality factor Q of the waveguides. Let us consider the limiting case $Q \to \infty$, which can be realized by filling the waveguide with an amplifying active medium that compensates for the losses in the waveguide. An example of such an analyzer is an active Fabry–Perot interferometer used for measuring laser spectra ([1,5]). In this case, for arbitrarily ...

For large l'_{gr}, attenuation in the waveguide is absent, and the beam-expansion coefficient is $a = 1$. It follows from (3) that in this case, for very large group-velocity retardations ($l'_{\text{gr}} \to \infty$), the resolving power also increases without bound. It should be borne in mind, however, that (3) was obtained on the assumption that, within the beam width β, the angular-frequency sensitivity remains constant. If the change of θ within the angle β, which in the present case is substantial, is taken into account, we arrive at the following expression for the limiting resolving power:

$$R(Q \to \infty,\ l'_{\mathrm{gr}} \to \infty) = 2(L/\lambda)^2. \tag{7}$$

The relation obtained gives the maximum attainable resolving power of a spectrum analyzer of given length, corresponding to infinitely large values of the group retardation and quality factor in the feeding waveguides. A higher resolving power cannot be achieved in multibeam interferometers with spatial scanning of the spectrum under any conditions.

The proposed general (waveguide) theory of multibeam interferometers has made it possible to consider, from a unified point of view, various spectrum analyzers. A general expression has been obtained for the resolving power provided by the method of multibeam interferometry.

The analysis carried out has shown that, in attempting to increase the resolving power of multibeam interferometers by increasing the length of the radiating aperture, and also by using modes with large group retardation, or both simultaneously, we arrive at a limit determined by the heat losses in the system and equal to the quality factor of the waveguides used.

The theory developed does not touch on questions of the influence of manufacturing accuracy on resolving power. Naturally, in many practical cases it is precisely the manufacturing accuracy, and not losses in the feeding system, that limits the resolving power. It should be noted, however, that the general waveguide theory of multibeam spectrum analyzers presented above can be developed for determining the limitations on resolving power due to manufacturing inaccuracies of the system, which lead to an increase in scattering losses equivalent to heat losses.

Peoples' Friendship University
named after Patrice Lumumba

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Note: Figure translations are in progress. See original paper for figures.