THE STRUCTURE OF THE FIELD OF A TROPICAL HURRICANE

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Abstract

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The importance of calculating the paths followed by tropical hurricanes is beyond dispute: they pose a danger even to large modern ships and every year cause great damage to the countries that lie in their path. However, in the extensive literature devoted to this phenomenon there is no theoretical explanation of the relative constancy of the prevailing tracks along which the "eye of the hurricane" moves, nor is there a solution of the corresponding complex thermo-hydrodynamic problem in its complete formulation. One thing is beyond doubt: such constancy is caused by the **thermodynamics** of a tropical hurricane, which, in our terminology, should be regarded as a "heat engine of the fifth kind," operating in the atmosphere (we have dealt earlier with four kinds of "heat engines in the atmosphere" ([1])). The heater for a "heat engine of the fifth kind," as a rule, is provided by the surface waters of the ocean, which have a particularly high temperature—the waters of warm currents in the Gulf Stream, Kuroshio, and other systems.

This hypothesis is confirmed not only by the attraction of hurricane tracks toward the axes of these currents, but also by two further circumstances known to all researchers: the complete absence of hurricanes of this type in the southern part of the Atlantic Ocean (regardless of season), connected with the absence of analogous "guiding" warm streams, and the complete absence of hurricanes ever recorded in winter, from January to March, in the North Atlantic and Pacific Oceans.

To construct a quantitative thermo-hydrodynamic theory that takes these circumstances into account, it is first of all necessary to outline at least an approximate sketch of the structure of the field of a tropical hurricane; such a task is solvable on the basis of the numerous existing studies.

In particular, very valuable quantitative data are contained in the work of E. Palmen and G. Riehl ([2]), who analyzed the results of direct measurements of wind speeds in hurricanes. On the basis of these data, with increasing distance from the eye of the hurricane, the tangential component of the wind velocity may be considered to decrease according to a logarithmic law. Let us write this law, taking as the argument, instead of the radius vector measured from the center of the hurricane, the dimensionless argument r/r_0, where r_0 denotes the radius of the hurricane eye. Then for the tangential component v we obtain

$$v = A - B \ln r/r_0. \tag{1}$$

Because of the difficulty and danger of making measurements in the immediate vicinity of the "eye," researchers confine themselves to the study of velocities no nearer than 2° of meridian from the center of the hurricane, while Palmen and Riehl extrapolated their relation to 1° of meridian, i.e., to 111 km from the center. Let us test the validity of extrapolation to an even closer distance from the center: to the boundary of the "eye" itself, whose radius, on the basis of numerous materials ([3], [4]), we shall take as $r_0 = 15$ km. At the outer boundary of the hurricane, for $r = R$, put $v = 0$. Then it becomes possible to write

$$B = A/(\ln R/r_0). \tag{2}$$

For the radial component u and the tangential component v, let us write the Navier–Stokes equations in a cylindrical coordinate system. The velocity of translational motion of the entire hurricane system is very small in comparison with the wind velocities in it, and sometimes even becomes equal to zero.

Therefore, at each point of the field we shall consider the motion to be steady and set the time derivatives of u and v equal to zero:

$$\frac{1}{\delta}\frac{\partial p}{\partial r} = -2\bar{\omega} v - \frac{v^2}{r} + \nu \frac{\partial^2 u}{\partial z^2}; \tag{3}$$

$$\frac{1}{\delta}\frac{\partial p}{r\,\partial \psi} = 2\bar{\omega} u + \frac{uv}{r} + \nu_r \frac{\partial^2 v}{\partial r^2} + \nu \frac{\partial^2 v}{\partial z^2}. \tag{4}$$

Here p is pressure; δ is air density; r is the distance from the center; ψ is the azimuth of the point; $\bar{\omega} = \omega \sin \varphi$ (ω is the angular velocity of the Earth' s rotation, φ is the latitude of the place); ν_r is the kinematic viscosity that must be taken into account when considering the friction of coaxial cylindrical layers*; ν is the kinematic viscosity usually taken into account when considering friction between horizontal air layers. We shall regard the hurricane field as axisymmetric, as in most theoretical works. Then it will be legitimate to set the left-hand side of (4) equal to zero.

Let us begin the analysis with the surface on which the pressure in the undisturbed field becomes equal to 950 mb, instead of the pressure of 1000 mb assumed here at ocean level. It is precisely here that the maximum of the component u is clearly seen, as well as the attainment of an interesting constancy of the component v on the epures in work (2).

In turn, ν can be expressed on the basis of the known semiempirical theory of turbulence,

$$\nu = k^2 \frac{(\partial u/\partial z)^3}{(\partial^2 u/\partial z^2)^2},$$

where k is the so-called Karman constant. Consequently:

$$\nu \frac{\partial^2 u}{\partial z^2} = k^2 \frac{(\partial u/\partial z)^3}{(\partial^2 u/\partial z^2)}. \quad (5)$$

On the selected surface $\partial u/\partial z = 0$, while in the denominator of the fraction $\partial^2 u/\partial z^2 \neq 0$. Hence the last term on the right-hand side of (3) may be set equal to zero, and instead of this equation we may write

$$\frac{1}{\delta}\frac{\partial p}{\partial r} = -2\bar{\omega} v - \frac{v^2}{r}. \quad (6)$$

Let us substitute into (6) the expression for v from (1) and integrate the resulting equation from r_0 to r. For existing hurricane parameters, the contribution of the integral of the first term on the right-hand side of (6) is negligibly small in comparison with the contribution of the second term, and begins to have some effect only at the periphery of the hurricane region. Discarding the negligibly small term, we obtain

$$p - p_0 = \delta A^2 \left[\ln\frac{r}{r_0} - \frac{B}{A}\left(\ln\frac{r}{r_0}\right)^2 + \frac{1}{3}\left(\frac{B}{A}\right)^2\left(\ln\frac{r}{r_0}\right)^3 \right]. \quad (7)$$

This is the equation of the curve of the barometer fall toward the center of a hurricane, obtained theoretically for the first time. As an example, let us specify the following characteristics of a hurricane: $r_0 = 15$ km, $R = 600$ km; consequently, $R/r_0 = 40$; $A = 75$ m/sec and, on the basis of (2), $B = 20.3$ m/sec. Then calculations by (7) give the barometer fall toward the center of the hurricane shown by curve 1 in Fig. 1. Along the abscissa axis are plotted the distances r from the hurricane center, and downward along the ordinate axis –the fall of atmospheric pressure relative to the region outside the hurricane, expressed in millibars.

Curve 1 closely resembles actual pressure records in hurricanes obtained on barographs. But the similarity is not confined to the general form of the curve. The law of the extrapolated dependence (1) is also confirmed by the value of the absolute pressure minimum calculated theoretically. Indeed, substitute $r = R$ into (7). Then on the left-hand side of (7) there arises the difference Δp between the pressure outside the hurricane and the pressure at the boundary of the eye. Performing simple transformations on the right-hand

Fig. 1

Figure 1: Fig. 1

* Strictly speaking, cylinders with helical guides.

part, on the basis of (2) we find

$$\Delta p = \frac{\delta}{3} A^2 \ln \frac{R}{r_0}. \tag{8}$$

This relation may be rewritten as follows:

$$A = \sqrt{\frac{3}{\delta \ln R/r}} \sqrt{p}. \tag{9}$$

Substituting here the numerical values of the quantities under the first radical, and taking, on the basis of the ellipse from (2), that the maximum component A at ocean level is approximately 10% smaller than on the 950-millibar surface investigated, we then find that at ocean level

$$v_{\text{max}} = 7.3\sqrt{\Delta p} \text{ m/sec.} \tag{10}$$

Fig. 1

This theoretical relation is very close to the empirical formula obtained by R. Kraft ([5]) on the basis of an analysis of maximum pressure falls in 14 hurricanes. Let us rewrite his formula, expressing wind speeds not in knots, as in Kraft' s work, but in meters per second,

$$V_{\text{max}} = 7.2\sqrt{1013 - p_{\text{center}}} \text{ m/sec.} \tag{10'}$$

True, here V denotes the total magnitude of the velocity vector, and not the tangential component, which is close to it in value. On the other hand, Kraft measures the pressure fall not from the value existing beyond the hurricane region, but from an arbitrarily chosen maximum value: from the normal pressure of 1013 mb.

The good agreement between (10) and (10′) shows that on the dashed part of curve 2 in Fig. 1, constructed from (7), the fall of v to zero toward the calm region of the "eye" in fact occurs very close to its boundary. It is natural to suppose that equation (4), not yet used, makes it possible to find the law of variation of the radial component u as a function of r. In an axisymmetric hurricane on the 950-millibar surface, the ellipse from (2) permits us to set equal to zero both the left-hand side of (4) and the last term on the right-hand side, writing as a result

$$2\bar{\omega}u + uv/r + \nu_r \partial^2 v/\partial r^2 = 0. \tag{11}$$

We shall try to take account of lateral friction between layers following the example of (5), replacing derivatives with respect to z by derivatives with respect to r. On the basis of (1), $\partial v/\partial r = -B/r$ and $\partial^2 v/\partial r^2 = B/r^2$. Consequently,

$$\nu_r \frac{\partial^2 v}{\partial r^2} = k^2 \frac{(\partial v/\partial r)^3}{\partial^2 v/\partial r^2} = -k^2 \frac{B^2}{r}. \tag{12}$$

Thus, instead of (11) one must write

$$2\bar{\omega}u + uv/r - k^2 B^2/r = 0. \tag{13}$$

It follows from this that

$$u = k^2 B^2/(2\bar{\omega}r + v). \tag{14}$$

This relation withstands a test near the outer boundary of the hurricane, where the velocities are small. For example, at $r = 500$ km curve 2 gives $v = 4$ m/sec. At latitude $\varphi = 30^\circ$, (14) leads to the expression $u = 10.2\,k^2$. The angle between the tangent to the isobar and the wind-velocity vector here may be taken as $a = 18^\circ$. Hence, $\operatorname{tg} a = u/v = 0.325$ and, on the basis of (14), $k = 0.36$. The order of magnitude is plausible, although under natural conditions one should expect $k < 0.36$.

Unfortunately, in the most interesting part of the hurricane the kinematic viscosity in fact apparently exceeds the value v_r following from the Prandtl–Kármán theory, and (14) is not applicable there. A number of works quite rightly indicate that any reliable measurement of u is impossible there. However, on the basis of the same works and on the basis of modern photographs of cloudiness in hurricanes from artificial Earth satellites, it may be assumed that the angle α retains the value noted above over a large extent of the hurricane field. Therefore, in Fig. 1 let us plot curve 3, representing the increase of u as the hurricane center is approached, proceeding from an equation analogous to (1): $u = a - b\ln r/r_0$. For the specified values $\alpha = 18^\circ$, $a = 24.4$ m/sec, $b = 6.6$ m/sec. Proceeding from the form of curve 3, one may attempt to find the law of variation of the vertical component of the wind velocity on the 950-millibar surface. It cannot yet be written in the usual form of the continuity equation, owing to the unreliability of data on the vertical distribution of u. We shall proceed otherwise. Let us assume that for a layer of height h the amount of air flowing in from outside through a cylindrical boundary surface of radius r is equal to the amount of air flowing inward through the surface of radius $r - dr$, and of air going along the vertical axis z with velocity w through a ring of width dr. This continuity condition is written in the form

$$w = \frac{h}{r}\frac{d}{dr}(ur). \tag{15}$$

Substituting here the two-term expression for u given above, after elementary transformations we obtain:

$$w = 100\,\frac{h}{r}(u - b)\ \text{cm/sec}. \tag{16}$$

The numerical factor 100 appears because w is expressed here in centimeters per second, unlike u and v, which were expressed in meters per second.

Curve 4 in Fig. 1 represents the variations of w under the condition $h = 500$ m. As we see, at a distance of 225 km from the hurricane center, 4 crosses the axis of abscissas, and its ordinates become negative. This means that a descending motion of air exists here. Let us note that over the corresponding extensive ring in the hurricane region, observations reveal a decrease in the relative humidity of the air from 95% outside the hurricane to 85% (see, for example, Fig. 10 in work (3)). The reason has become clear: air descends which has lost water that fell out in torrential precipitation nearer the eye of the hurricane and, in addition, has reduced relative humidity as a result of adiabatic compression during descent. Curves 3 and 4 should in reality behave differently than in the drawing on their dashed portions. Curve 3 should drop rapidly to the axis of abscissas near the boundary of the hurricane eye, indicating the onset of calm. Curve 4 here should reflect the upward outflow of all the air moving toward the eye along the radii r. Further investigations will undoubtedly make it possible to refine the course of curves 3 and 4 in the vicinity of the boundary of the hurricane eye. The form of curve 2 will change least of all (instead of the dashed-segment portion).

It is very important to find the reason for the discrepancy between the semiempirical theory of turbulence and the conditions existing here: in its present state, on the basis of (14), the theory would lead to greatly underestimated values of u (of the order of 1.5 m/sec) in these regions.

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Note: Figure translations are in progress. See original paper for figures.