ON ELECTRIC FIELDS IN THE EARTH' S MAGNETOSPHERE

GEOPHYSICS

1969

SovietRxiv

Abstract

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UDC 551.594

GEOPHYSICS

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(Presented by Academician S. N. Vernov, 2 VI 1969)

1. One of the central problems in the physics of the magnetosphere is to clarify the nature of the electric fields associated with geomagnetic disturbances. In the ionosphere such fields are concentrated along the boundaries of the polar caps and excite in each hemisphere two narrow current (probably Hall) jets directed toward one another from the dayside to the nightside. These currents are partly closed through low-latitude regions, which indicates penetration of the field into the radiation belts. Experiment shows that the excitation of the current jets is associated with a significant disturbance of the plasma in the magnetosphere and leads, in particular, to the blowing-out of cold plasma and to the injection into the inner belt of particles with energies $\lesssim 10$ keV (up to ≈ 300 keV during the most powerful storms).

The present state of the question and the principal hypotheses concerning the nature of electric fields are set out in detail in ([1]), Ch. 10. In the present note a new physical effect is indicated which makes it possible to explain a number of characteristic features of electric fields in the magnetosphere. As satellite measurements have shown, the disturbances under consideration are associated with the formation, in the region of the radiation belts, of a longitudinally asymmetric hot plasma ring and with its symmetrization ([2]).

Let us consider the dynamics of this process under the following assumptions: a) the asymmetric disturbance is small in comparison with the symmetric distribution of the main mass of particles; b) the plasma pressure gradient is small in comparison with the gradient of the magnetic pressure of the dipole field; c) the conductivity along the lines of force in the ionosphere and above is infinite; d) the Hall conductivities σ_x and Pedersen conductivities σ_Π are nonzero only in a thin spherical layer (the E-layer of the ionosphere) and do not depend on latitude or longitude; e) the plasma is located on lines of force entering the ionosphere at high latitudes; f) the duration of the processes under consideration is such that the magnetic moment and the invariant of longitudinal action are conserved.

In this case the problem is linear and is formulated according to the scheme proposed by Chang, Pearlstein, and Rosenbluth for the analysis of the stability of belts ([3]). Hot particles (which, for simplicity, we take to be protons) are described by the continuity equation, taking into account magnetic drift in the dipole field and electric drift in the self-consistent field with potential U. By virtue of ideal conductivity, U is constant along the lines of force. Cold electrons do not experience magnetic drift, and quasineutrality is ensured by currents along the lines of force, closing through the ionosphere.

The geometrical coordinates in the magnetosphere are the parameter L (the distance from the apex of a line of force to the center of the Earth, measured in Earth radii a) and the longitude φ. In the ionosphere, instead of L, it is convenient to introduce the polar angle θ, measured from the dipole axis. According to the equation of a line of force, $L\sin^2\theta = 1$. In the dipole field the majority of the particles of the plasma ring are concentrated near the equatorial plane, which substantially simplifies the problem. As shown in ([4]), § 4, for particles with reflection points at low and middle latitudes the approximate invariants are

$\mu = \mathcal{E}L^3$ ($\mathcal{E}$ is the particle energy) and the angle α between the velocity and the field line at the equator.

The velocity of the magnetic drift practically does not depend on α. The rate of change of L and φ due to the electric drift, according to (3), for $U = U(L, \varphi)$, also does not depend on α.

Therefore it is sufficient to consider the distribution function giving the number of particles in a tube with unit cross section in the equatorial plane and in a unit interval of μ. Let $n_0(L, \mu)$ denote the unperturbed distribution, and let $n'(L, \mu, \varphi, t)$ be a small asymmetric perturbation.

The continuity equation

$$\frac{\partial n'}{\partial t} + \Omega_M(L,\mu)\frac{\partial n'}{\partial \varphi} + \frac{\partial}{\partial \varphi}\left[\Omega_E(L, M, \varphi, t)n_0\right] + \frac{1}{L}\frac{\partial}{\partial L}\left[L\dot{L}_E(L,\mu,\varphi,t)\right] = 0 \quad (1)$$

is reduced to the form

$$\frac{\partial n'}{\partial t} + \Omega_M(L,\mu)\frac{\partial n'}{\partial \varphi} + \frac{c}{H_0 a^2}\frac{\partial U}{\partial \varphi}\frac{1}{L}\frac{\partial}{\partial L}L^3 n_0 = 0, \quad (2)$$

where

$$\Omega_M = \frac{3c\mu}{eH_0 a^2 L^2}; \qquad \Omega_E = -\frac{cL^2}{H_0 a^2}\frac{\partial U}{\partial L}; \qquad \dot{L}_E = \frac{cL^2}{H_0 a^2}\frac{\partial U}{\partial \varphi} \quad (3)$$

are the angular velocities of the magnetic and electric drifts in φ and the rate of change of L in the electric field; H_0 is the field at the Earth' s equator.

The absence of magnetic drift for cold electrons is compensated by currents along the field lines, and from the quasineutrality condition

$$j_{\|} = -eL^3 \int \Omega_M \frac{\partial n'}{\partial \varphi}\, d\mu, \tag{4}$$

where $j_{\|}$ is the current density at the boundary of the ionosphere. The factor L^3 is due to the fact that the cross section of the tube at the Earth (at high latitudes) is $2L^3$ times smaller than in the equatorial plane, and the currents flow from both hemispheres.

At high latitudes the magnetic field in the ionosphere is radial, and the equation $\operatorname{div} \mathbf{j} = 0$ has the form

$$\operatorname{div}\{\sigma_\Pi \nabla U + \sigma_X [\mathbf{e}_R \nabla U]\} = \frac{\partial j_{\|}}{\partial R}. \tag{5}$$

Integrating (5) over the thickness of the ionosphere and taking into account that, by assumption, σ_Π, σ_X, and U do not depend on R, with (4) we obtain

$$\frac{1}{\theta}\frac{\partial}{\partial \theta}\,\theta\frac{\partial U}{\partial \theta} + \frac{1}{\theta^2}\frac{\partial^2 U}{\partial \varphi^2} = -\frac{ea^2L^3}{\sigma_\Pi h}\int \Omega_M \frac{\partial n'}{\partial \varphi}\, d\mu \tag{6}$$

(for $\theta \ll 1$, $\sin\theta \approx \theta$).

We shall seek solutions localized in the high-latitude regions (bounded at $\theta = 0$ and decreasing for large θ).

Let us consider processes with a characteristic time much shorter than the period of the longitudinal drift of protons, and omit in (2) the term with Ω_M. We seek the solution in the form $\exp(\gamma t - im\varphi)$. From (2) we find

$$n' = \frac{imc}{H_0 a^2 \gamma}\,\frac{U_m(L)}{L}\,\frac{\partial}{\partial L}(L^3 n_0), \tag{7}$$

and from (6)

$$\frac{1}{\theta}\frac{\partial}{\partial \theta}\,\theta\frac{\partial U_m}{\partial \theta} - \frac{m^2}{\theta^2}U_m = \frac{3m^2c^2}{\sigma_\Pi \gamma h H_0^2 a^2}\,U_m \frac{\partial}{\partial L}\left[L^3 \int n_0 \mu\, d\mu\right]. \tag{8}$$

For an isotropic angular distribution of particles, the integral in (8) is expressed through the pressure $p(L)$ and is equal to $\approx aL^4 p(L)$.

2. By the substitution $U_m = \theta^{1/2}\psi_m$, (8) is reduced to the form of the Schrödinger equation with zero energy and potential

$$\Phi = -\frac{m^2 - \frac{1}{4}}{\theta^2} + \frac{3m^2c^2}{\sigma_\Pi \gamma H_0^2 ah}\frac{d}{dL}[L^7 p(L)]. \tag{9}$$

It is easy to see that if Kadomtsev' s stability condition (5) for a dipole field, $[L^7 p(L)]' > 0$, is violated even locally, then convection is excited. Indeed, one can then find sufficiently small positive values of γ for which, in the region $[L^7 p(L)]' < 0$, Φ has the form of a potential well separated from infinity by an impenetrable barrier. For large m the well is quasiclassical, and the spectrum γ is determined by Bohr' s quantization formula.

Let us consider the evolution of an asymmetric ring with $p \sim L^{-7}$ at large L. Put

$$p(L) = \begin{cases} p_0 \left(\frac{L}{L_0}\right)^{-7} & (L > L_0), \\ p_0 \left(\frac{L}{L_0}\right)^{\nu} & (L \le L_0). \end{cases} \tag{10}$$

Then (8), for $\theta < \theta_0 = L_0^{-1/2}$, has the trivial solution $U_m = A(\theta/\theta_0)^m$, and for $L > \theta_0$ takes the form

$$\frac{\partial^2 U_m}{\partial \theta^2} + \frac{1}{\theta}\frac{\partial U_m}{\partial \theta} + \left[\lambda\left(\frac{\theta_0}{\theta}\right)^{2(\nu+6)} - \frac{m^2}{\theta^2}\right] U_m = 0; \quad \lambda = \frac{3m^2c^2(\nu+7)p_0 L_0^6}{\sigma_\Pi |\gamma| H_0^2 ah}. \tag{11}$$

The sought solution is

$$U_m = BJ_{m/(\nu+5)}\left[\frac{\theta_0\sqrt{\lambda}}{\nu+5}\left(\frac{\theta_0}{\theta}\right)^{\nu+5}\right], \tag{12}$$

where J is the Bessel function. For $\theta \gg \theta_0$, $U_m \sim \theta^{-m}$. The spectrum of the values of λ is determined from the continuity of U_m and U_m' at $\theta = \theta_0$.

Let us consider the harmonics with $m = 1$ (in this case the field penetrates most effectively into the region of the radiation belts and exists the longest). The characteristic equation has the form

$$(\nu+5)xJ'_{1/(\nu+5)}(x) + J_{1/(\nu+5)}(x) = 0. \tag{13}$$

For $x \ll 1$, $J_{1/(\nu+5)} \sim x^{1/(\nu+5)}(1 - x^2/4)$. Hence we find the zero root of (13):

$$\lambda_0 = 4(\nu+5)/\theta_0^2. \tag{14}$$

The corresponding eigenfunction has no nodes and does not give concentrations of the electric field. The remaining roots correspond to $x \gtrsim 1$, and for large ν the Bessel functions are close to $J_0(x)$. In this case the roots x_n are close to the roots of $J_1(x)$. Thus, for $n \geq 1$,

$$\lambda_n = \frac{(\nu+5)^2}{\theta_0^2}\, x_n^2.$$

All nodes of the corresponding functions are concentrated in the narrow region θ onto which the inner boundary of the plasma ring is projected. Therefore, in the corresponding regions of the ionosphere, electric fields and currents must be concentrated. The first harmonic gives a jump of the potential in this region and forms Hall current jets, while higher harmonics produce the fine structure of these jets.

We note that the zero harmonic decays much faster than the others. Let us compare γ_0 and γ_1:

$$\gamma_0/\gamma_1 = 4(\nu+5). \tag{15}$$

For probable values $\nu \sim 5 \div 10$, the ratio is ~ 50. Therefore the zero harmonic, which decays rapidly and does not give concentrated current systems, is difficult to observe. As $\sigma_\Pi \to 0$, $\gamma_n \to \infty$. This is connected with the neglect, in the initial equations, of charge separation due to inertial forces. The corresponding criterion is easily obtained by comparing $\Omega_M\, \partial n'/\partial\varphi$ with the inertial terms

$$\frac{Mc^2\gamma}{ea^2H^2}\,\frac{\partial}{\partial\varphi}\left[\frac{n_0}{L}\frac{\partial U}{\partial L}\right] \quad \text{and} \quad \frac{Mc^2\gamma}{ea^2H^2}\,\frac{\partial}{\partial L}\left[n_0\frac{\partial U}{\partial\varphi}\right].$$

Using (7) and (3), we find the condition

$$\gamma \ll V_\mathrm{T}/aL, \tag{16}$$

where M and V_T are the mean mass and thermal velocity of the protons. On the other hand, for $\sigma_\Pi \to \infty$, $v_n \to 0$. Let us recall that the results are valid under the condition $\gamma \gg \Omega_M$.

Thus, the gradient of the concentration of fast particles along the trajectories of magnetic drift in a trap with conducting ends induces currents and electric fields that sharply accelerate the equalization of concentration. This effect probably plays a major role in the physics of the magnetosphere and, in particular, may lead to the rapid penetration of solar-wind plasma from the boundary layer into the tail.

3. The specific dynamics is determined by the initial conditions. Suppose that at $t = 0$ the region of maximum pressure has the form (in the equatorial plane) of an eccentric circular ring, closer to the Earth on the nightside (such a configuration may arise during rapid expansion of the magnetosphere [4]). It is easy to see that, as a result of magnetic drift, a double charged layer with a potential jump $\delta U \sim \cos\varphi$ will arise on this ring, and Hall current jets will be induced in the ionosphere.

Quantitative estimates are meaningful only in order of magnitude, since our model does not take into account, for example, the diurnal variation of conductivity. From (15) and (7) we have:

$$\gamma_1 = -\frac{9(\nu+7)}{64\pi\nu(\nu+5)^2}\,\frac{\beta_0 c^2}{\sigma_\Pi a L_0 h}; \qquad \delta U \approx \frac{9(\nu+7)}{64\pi\nu(\nu+5)^2}\,\frac{acH\beta_0}{L_0^2\sigma_\Pi h}\,\frac{\delta p}{p_0}, \qquad I \approx \delta U h \sigma_X \tag{17}$$

($\beta_0 = 8\pi|\nabla p|/|\nabla H^2|$ at $L = L_0$, δp is the asymmetric disturbance of pressure). For $\beta = 1$, $\delta p \sim p$, $\nu = 6$, $L_0 = 5$, $h = 2\cdot 10^6$ cm, $\sigma_\Pi = 3\cdot 10^5$ CGSE and $\sigma_X = 4\sigma_\Pi$, we obtain $\gamma \approx 3\cdot 10^{-4}$ sec^{-1}, $\delta U \approx 2\cdot 10^4$ V, $I \approx 5\cdot 10^4$ A. The perturbation of the magnetic field under the jet is $\sim 10^{-3}$ oersted. These parameters correspond to the characteristics of most polar disturbances, and it is natural to suppose that the effect considered is the cause of the generation of such electric fields. It follows from (17) that the work of the currents W depends only on the energy of the plasma and, in order of magnitude, is $\delta p \cdot w$ (w is the effective volume of the ring). The parameter values given yield $W \approx 10^{21}$ erg.

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Received
19 V 1969

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Note: Figure translations are in progress. See original paper for figures.