GREEN’ S FUNCTIONS IN THE THEORY OF FERROMAGNETIC RESONANCE

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Abstract

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GREEN' S FUNCTIONS IN THE THEORY OF FERROMAGNETIC RESONANCE

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The paper gives a method for constructing an approximate solution of the chain of equations for two-time Green' s functions, making it possible to obtain expressions for the damping of homogeneous precession, as well as for the linewidth and line shape in ferromagnetic resonance. The method is based on the approximation of many-particle functions by functions of lower order in accordance with the ideas and methods of Bogolyubov (^{1}).

Let us consider a system of interacting magnetic moments situated in constant and high-frequency magnetic fields. The Hamiltonian of the system has the form:

$$\mathcal{H} = \mathcal{H}_0 + \mu h S^x e^{i\omega t}; \tag{1}$$

$$\mathcal{H}_0 = \mu H \sum S_f^z - \frac{1}{2} \sum I(f_1, f_2) S_{f_1} S_{f_2} - \frac{1}{2} \sum \delta I(f_1, f_2) S_{f_1}^z S_{f_2}^z + \mathcal{H}_{\text{demag}}; \tag{2}$$

$$\mathcal{H}_{\text{demag}} = \frac{\mu^2}{2N} N_\perp \sum S_{f_1}^+ S_{f_2}^- + \frac{\mu^2}{2N} N_z \sum S_{f_1}^z S_{f_2}^z. \tag{3}$$

Here H is the constant magnetic field; h is the amplitude of the high-frequency field; S are spin operators; f is a lattice-site vector; $N_\perp$ and N_z are demagnetizing factors; I is the energy of the isotropic exchange interaction; δI is the energy of the anisotropic interaction.

The Hamiltonian of the system has exact eigenstates: the ground state—with magnetization $m = -\mu S$—and an excited state—with one spin wave. The energy of this elementary excitation is

$$E(k) = \mu H + [\bar{I} - J(k)]S + \delta\bar{I} \cdot S + [\mu^2 N_\perp \Delta(k) - \mu^2 N_z]S. \tag{4}$$

The condition of positivity of $E(k)$ for all values of k is the criterion for the stability of the ground state. This imposes a restriction on the values of the constant magnetic field, which are determined by the inequality

$$\mu H + \delta\bar{I} \cdot S - \mu^2 N_{zS} > 0, \qquad \delta\bar{I} = \sum_{f'} \delta I(f, f'). \tag{5}$$

We shall seek a solution for the function

$$G_1(k) = \sum_{f'} \langle\langle S_f^+(t) \mid S_{f'}^-(t')\rangle\rangle e^{-i(f-f',k)} \quad \text{for } k = 0, \tag{6}$$

since the components of the high-frequency susceptibility tensor are expressed through it (^{2}).

The function G_1 is calculated exactly at $T = 0$ (when the system is in the ground state). In this case we have

$$G_1(k, \omega) = \frac{i}{2\pi} \frac{2S}{\omega - E(k)}. \tag{7}$$

Thus, damping in the system at $T = 0$ is absent. Substituting (6)

in the formula for the high-frequency susceptibility (2), we obtain

$$\chi_{xx} = \frac{\mu^2 S}{2} \left[\frac{1}{\omega - E(0)} - \frac{1}{\omega + E(0)}\right]; \qquad \chi_{yx} = -\frac{\mu^2 S i}{2} \left[\frac{1}{\omega - E(0)} + \frac{1}{\omega + E(0)}\right]. \tag{8}$$

We see that at $T = 0$ the resonance line has a δ-shaped form. At nonzero temperature the equation for $G_1(0)$ has no exact solution; one must consider the chain of equations obtained by successive differentiation of the Green functions (3). The first equation of this chain has the form

$$[\omega - \mu H - \sigma\delta\bar{I} - \mu^2\sigma(N_\perp - N_z)]G_1(0) =$$

$$= \frac{i}{2\pi} 2\sigma + \sum_{f'} \delta I(f, f')\langle\langle S_f^+ \nu_{f'} \mid S^-\rangle\rangle + \frac{1}{N}\mu^2(N_\perp - N_z) \sum_{f'} \langle\langle S_f^+ \nu_{f'} \mid S^-\rangle\rangle, \tag{9}$$

where $\nu_f = S_f^z + \sigma,\ \sigma = -\langle S_f^z\rangle$.

If the last two terms in (8) are discarded, we obtain the solution in the so-called self-consistent-field approximation. Such a solution was obtained in [2] in the absence of anisotropy. In our case this solution has the form

$$G_1(0) = \frac{i}{2\pi}\,\frac{2\sigma}{\omega - \mu H - \sigma\delta\bar{I} - \sigma\mu^2(N_\perp - N_z)}. \tag{10}$$

Using the principle of weakening of correlations, it is not difficult to show that in the absence of anisotropy the solution (10) is asymptotically exact (as $N \to \infty$). The influence of the demagnetizing field appears in a change of the resonance frequency, but not in a broadening of the line.

In the presence of anisotropy, we must deal with an infinite chain of coupled equations. Let us note that the second Green function has different kinematic properties for $f \neq f'$ and for $f = f'$. For example, for $S = 1/2$ the second function for $f = f'$ reduces to the first. To take this circumstance into account, we write an equation for the function

$$G_2(k) = \sum_{f'} (1 - \Delta_{ff'})\langle\langle S_f^+ \nu_{f'} \mid S^- \rangle\rangle e^{-i(f-f',\,k)}. \tag{11}$$

The equation for $G_2(k)$ will contain functions of the form

$$\langle\langle S_f^+ \nu_{f'} \nu_h \mid S^- \rangle\rangle \quad \text{and} \quad \langle\langle S_f^+ S_{f'}^- S_h^+ \mid S^- \rangle\rangle. \tag{12}$$

Their properties also depend on whether the indices of the lattice sites coincide. Let us first consider the case $S = 1/2$. Then, if at least a pair of indices coincide, the functions (12) reduce to functions of lower order. For functions with different indices we make an approximation. For this purpose let us consider the correlation functions $\langle S_f^+ S_{f'}^- S_h^+ \rangle$ and $\langle S_f^+ \nu_{f'} \nu_h \rangle$, where the averaging is over the full Hamiltonian. We represent these functions in the form

$$\langle S_f^+ S_{f'}^- S_h^+ \rangle = \langle S_f^+ S_{f'}^- \rangle \langle S_h^+ \rangle + \langle S_h^+ S_{f'}^- \rangle \langle S_f^+ \rangle + \Phi + \langle \Delta S_f^+ \Delta S_{f'}^- \Delta S_h^+ \rangle,$$

$$\langle S_f^+ \nu_{f'} \nu_h \rangle = \langle \nu_{f'} \nu_h \rangle \langle S_f^+ \rangle + \langle \Delta S_f^+ \nu_{f'} \nu_h \rangle. \tag{13}$$

If we neglect the last terms, then, to accuracy $O(h)$, we shall have

$$\langle\langle S_f^+ S_{f'}^- S_h^+ \mid S^- \rangle\rangle = \langle S_f^+ S_{f'}^- \rangle \langle\langle S_h^+ \mid S^- \rangle\rangle + \langle S_h^+ S_{f'}^- \rangle \langle\langle S_f^+ \mid S^- \rangle\rangle,$$

$$\langle\langle S_f^+ \nu_{f'} \nu_h \mid S^- \rangle\rangle = \langle \nu_{f'} \nu_h \rangle \langle\langle S_f^+ \mid S^- \rangle\rangle, \tag{14}$$

where the averages are now taken with the Hamiltonian $\mathcal{H}_0$. Approximations of this kind were made in [4]. Using (14), we obtain for $G_2(k)$ the integral equation

$$[\omega - E'(k)]\, G_2(k) - \sigma \frac{1}{N} \sum_{\nu} [J(\nu) - J(k-\nu) - 2\delta J(k-\nu)]\, G_2(\nu) =$$

$$= \frac{i}{2\pi} L(k) - A(k) G_1(0). \tag{15}$$

Here

$$E'(k) = \mu H + [\bar{I} - J(k)]\sigma + \delta I \cdot \sigma + \mu^2 \sigma [N_\perp \Delta(k) - N_z],$$

$$L(k) = \alpha(k) - 2\nu(k) - \langle S_f^+ S_f^- \rangle + 2\langle \nu_f^2 \rangle,$$

$$\begin{aligned} A(k) = \delta I \left[\gamma_k \nu(k) - \frac{1}{N} \sum \gamma_q \nu(q)\right] - \frac{1}{2} L(k)\, [\mu^2 N_\perp - I(\gamma_0 - \gamma_k)] \\ + I \left(1 - \frac{\gamma_k}{\gamma_0}\right) \sum \gamma_q \left[\frac{1}{2}\alpha(q) - \nu(q)\right] + \mu^2 \Delta(k) \left[\frac{1}{2}\alpha(0) N_\perp - \nu(0) N_z\right], \end{aligned}$$

$$\alpha(k) = \sum_f \langle S_f^+ S_{f'}^- \rangle e^{-i(f-f',\, k)}.$$

We shall solve equation (15) in the nearest-neighbor approximation. This makes it possible to represent the integral terms in the form

$$\frac{1}{N} \sum J(k-\nu) G_2(\nu) = I \frac{\gamma_k}{\gamma_0} \sum \gamma_\nu G_2(\nu).$$

After simplifying the integral terms, we can determine the quantity $\sum \gamma_k G_2(k)$, which enters into (9). As a result, for $G_2(k)$ we obtain the expression

$$\left[\omega - E'(0) - \frac{M_1}{1 - M_2}\right] G_1(0) = \frac{i}{2\pi} \left[2\sigma - \frac{\delta I}{1 - M_2} \frac{1}{N} \sum \frac{L(k)}{\omega - E'(k)}\right], \tag{16}$$

where

$$M_1 = \frac{\delta I}{N} \sum \frac{\gamma_k A(k)}{\omega - E'(k)}, \qquad M_2 = \frac{\sigma}{N} \sum \frac{\gamma_k}{\gamma_0} \frac{I(\gamma_0 - \gamma_k) - 2\delta I \cdot \gamma_k}{\omega - E'(k)}.$$

The quantity M_2 was obtained in considering Green' s functions with coinciding indices. For it we have the following estimate:

$$M_2' \ll 1, \qquad M_2'' \ll 1.$$

In accordance with these inequalities, M_2 may be neglected. With this degree of accuracy, the method developed also extends to $S > 1/2$, if the structure of the anisotropic term is similar to (2).

The quantity $M_1'(\omega)$ determines the correction to the frequency, and $M_1''(\omega)$ the damping of the homogeneous precession. These quantities will be increasing functions of the temperature. In the vicinity of the Curie point and in the paramagnetic region we shall have, for $M'(\omega)$, the estimate

$$M_1'(\omega) < \frac{(8I)^2}{\omega}\frac{T}{T_c}\frac{1}{36c}\sqrt{\frac{\dfrac{\mu H}{\sigma} + \delta I - \mu^2 N_z}{I}}, \tag{17}$$

where $c \sim 1.5$.

If the applicability of our method is considered in the spirit of perturbation theory, it is necessary to require that $M_1'(\omega) \ll E'(0)$. The quantity $M_1''(\omega)$ determines the shape of the resonance line. The imaginary part of the high-frequency susceptibility at the resonance point is also expressed through it. We give the value of M_1'' for a sphere ($N_\perp = N_z$) at $\omega = E'(0)$ in the vicinity of the Curie point and in the paramagnetic region:

$$M_1''(\omega_0) = \delta I\sqrt{\frac{\mu^2 N_z}{I}}\frac{T}{T_c}\frac{1}{2\pi^2}\frac{\delta I}{\mu H + \sigma\mu^2 N_z}. \tag{18}$$

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