RESULTS OF A STUDY OF A STATISTICAL MODEL OF THE EARTH’ S CRUST

GEOPHYSICS

1969

SovietRxiv

Abstract

Full Text

UDC 534.222.550.834.3

GEOPHYSICS

A. V. NIKOLAEV, F. S. TREGUB

RESULTS OF A STUDY OF A STATISTICAL MODEL OF THE EARTH' S CRUST

(Presented by Academician A. P. Vinogradov, 28 III 1969)

1. In Kazakhstan, along the Zhalanash—Taldy-Kurgan profile, the Ili Geophysical Expedition in 1965–1966 carried out deep seismic sounding work, consisting of continuous profiling over a 214 km base with four shot points on the profile and one remote source in Lake Balkhash (200 km from the profile); the spacing between instruments was 250 m. Wide-band apparatus, absolute calibration of the recording channel, and monitoring of the seismic energy of the sources made it possible to study in detail the amplitudes of elastic waves and to find the causes determining their spatial structure.

2. The wave pattern for all shot points is, in general outline, the same. At epicentral distances up to 6–15 km, several waves are traced in the first arrivals, having apparent velocities from 3.0 to 5.5 km/sec and visible frequencies of 9–12 Hz. At greater distances from the source a refracted wave is visible, which is traced throughout the entire observation interval (more than 200 km). Its predominant frequency is 7-8 Hz; the apparent velocity gradually increases with distance from the source from 5.8-6.0 to 6.2-6.5 km/sec. The wave field in subsequent arrivals has a complex interference character and unstable spatial correlation.

3. The true distribution of seismic parameters in the Earth's crust—the propagation velocities and absorption decrements of longitudinal and transverse waves—is so complex that the solution of the inverse problem is inevitably connected with a simplified description of the medium. It often turns out that several sections correspond to the same characteristic features of the wave field; their number increases sharply with increasing complexity of the model, i.e., with the growth in the number of parameters specifying the section. An increase in the detail of the investigations is accompanied by an increase in the ambiguity of the result, which may be regarded as a decrease in its reliability. This complexity of interpretation compels one to seek not the entire set of solutions, but only certain mean characteristics of the section, to which the mean characteristics of the wave field correspond. Complex fields of seismic parameters of the medium and

of elastic oscillations are conveniently described by statistical characteristics. The spatial distribution of any parameter is approximated by the sum of two fields: a deterministic one, which is a particular realization of the deterministic model adopted for interpretation, and a random one—homogeneous and isotropic within a certain region ([1]).

4. We assume the field of velocities of longitudinal waves $v(x, y, z)$ in the half-space $z > 0$ to consist of the following four components:

$$v(x, y, z) = \bar{v}(z) + v_n(x, y, z) + \delta v(x, y, z) + \Delta v(x, y),$$

where $\bar{v}(z)$ is the deterministic component—smoothly varying with depth and predominating in magnitude over the other terms; $v_n(x, y, z)$ is a large-scale inhomogeneity; $\delta v(x, y, z)$ is a realization of an isotropic and

locally homogeneous random field; $\Delta v(x, y)$ are inhomogeneities located near the daytime surface.

In a real medium, fluctuations of the longitudinal-wave velocity are necessarily connected with fluctuations of other seismic parameters. We assume that the change in the amplitudes of the first waves is determined only by the field $v(x, y, z)$, and we seek the relation between its components and the spatial structure of amplitudes and arrival times. The field of amplitudes of the first waves and the phase hodograph on the daytime surface can be represented in the following form ([2]):

$$\ln A(x, y) = \ln \bar{A}(r) + \ln A_n(x, y) + \ln \delta A(x, y) + \ln \Delta A(x, y),$$

$$T(x, y) = \bar{T}(r) + T_n(x, y) + \delta T(x, y) + \Delta T(x, y). \qquad (*)$$

Here $\ln \bar{A}(r)$ and $\bar{T}(r)$ are the components corresponding to the section $\bar{v}(z)$; r is the epicentral distance; $\ln A_n(x, y)$ and $T(x, y)$ are smoothly varying components corresponding to the inhomogeneity $v_n(x, y, z)$; $\ln \delta A(x, y)$ and $\delta T(x, y)$ are fluctuations corresponding to the field $\delta v(x, y, z)$; $\ln \Delta A(x, y)$ and $\Delta T(x, y)$ are distortions caused by the inhomogeneity $\Delta v(x, y)$ located near the observation surface.

The expediency of representing the amplitude field in the form $(*)$ is justified by the fact that small, fine-scale fluctuations of velocity also correspond to small, fine-scale fluctuations of amplitude, and the intensity $\ln \delta A$ is proportional to δv. This proportionality exists in the relations $v_n(x, y, z)$ and $A_n(x, y)$, $\Delta v(x, y)$ and $\Delta A(x, y)$ ([2]).

5. In observations on longitudinal profiles along the x-axis with several sources, we have a set of amplitude graphs and hodographs $\ln A^{(1)}(x), \ln A^{(2)}(x), \dots, T^{(1)}(x), T^{(2)}(x), \dots$. Their components have the

following properties ([3]): $\bar{A}^{(i)}(x)$ and $\bar{T}^{(i)}(x)$ have the same form for any source and are related by the relation $\ln \bar{A}^{(i)}(x) = C(x) + k\, d^2\bar{T}^{(i)}(x)/dx^2$, where $C(x)$ is a function decreasing with distance as $x^{-1/2}$; k is a constant factor; $\ln A_n^{(i)}(x)$ and $T_n^{(i)}(x)$ are not correlated, oscillate slowly, and their prevailing intervals of oscillation are approximately the same. The graphs $\delta T^{(i)}$ and $\ln \delta A^{(i)}(x)$ are related by the relation:

$$D \ln \delta A^{(i)}(x) = 4\pi f^2 D\delta T^{(i)}(x), \qquad D \ln \delta A^{(i)}(x) = \int_L g(x,y,z)\, dl = \bar{g}L, \quad (**)$$

where f is the frequency; D is the symbol of variance; $g(x,y,z)$ is the turbidity coefficient; $\bar{g}$ is its mean value for the predominant frequency f; L is the path of the wave along the ray over which the integral is calculated; $\ln \delta A^{(i)}(x)$ and $\delta T^{(i)}(x)$ have the same, comparatively small, correlation interval, determined by the dimensions of the fluctuations δv. The distortions of the amplitude graph and hodograph caused by surface inhomogeneities are correlated with one another, and the fluctuations of the logarithm of wave amplitudes and of the arrival times of their phases have the same order for different sources:

$$\ln \Delta A^{(i)}(x) \sim \ln \Delta A^{(j)}(x), \qquad \Delta T^{(i)}(x) \sim \Delta T^{(j)}(x).$$

This property is used to isolate fluctuations of amplitudes and times connected with surface inhomogeneities.

6. The problem of interpreting amplitude graphs and hodographs consists in the quantitative evaluation of the components of the velocity section: $\bar{v}(z)$ is established accurately, while for the remaining terms certain quantitative characteristics are determined that make it possible to regionalize and stratify the media studied: $v_n(x,z)$ is characterized by the mean square of the logarithm of amplitude, referred to the cube of the ray-path length; $D \ln A_n(x) : L^3 \sim v_n(x,z)$; $\delta v(x,z)$ is characterized by the spatial distribution of the turbidity coefficient $g(x,z)$, $\Delta v(x)$ by the mean fluctuations of the logarithm of amplitude $\ln \Delta A(x)$ and of the hodograph $\Delta T(x)$, calculated from several graphs.

7. Amplitude plots and hodographs consist of two parts: a high-frequency part, $\ln \delta A(x) + \ln \Delta A(x)$, $\delta T(x) + \Delta T(x)$, and a low-frequency part, $\ln \bar{A}(x) + \ln A_n(x)$, $\bar{T}(x) + T_n(x)$. For their separation, spatial filtering or weighted averaging is used. As the weighting function for low-frequency filtering, a normal curve is taken, possessing a number of good properties (4). The high-frequency component is determined as the difference between the observed and smoothed plots. The filtering parameters depend on the predominant intervals of fluctuations in the different components of the plot; in choosing them, a priori ideas about the medium are taken into

Fig. 1. Velocity section of the Earth' s crust. 1 $-\bar{v}(z)$; 2 —variation of the turbidity coefficient g with depth; isolines $-g \cdot 10^{-3}$ km^{-1} along the profile

Figure 1: Fig. 1. Velocity section of the Earth' s crust. 1 $-\bar{v}(z)$; 2 —variation of the turbidity coefficient g with depth; isolines $-g \cdot 10^{-3}$ km^{-1} along the profile

account. In separating small and large fluctuations, an averaging interval of the plots of 5 km was taken.

8. The deterministic component of the amplitude plot $\ln \bar{A}(x)$ was found by averaging the smoothed amplitude curves constructed

Fig. 1. Velocity section of the Earth' s crust. 1 $-\bar{v}(z)$; 2 —variation of the turbidity coefficient g with depth; isolines $-g \cdot 10^{-3}$ km^{-1} along the profile

as a function of epicentral distance. Comparison of the second derivative of the smoothed component of the hodograph with the plot $\ln \bar{A}(x)$ showed relatively weak agreement between them. The deterministic component of the velocity section of the medium is only faintly expressed in the characteristic features of the amplitude plots and hodographs. Its extraction against the background of noise caused by fluctuations of velocities in the upper part of the section is possible only in the roughest outline. Nothing indicates possibly existing boundaries of discontinuity in the consolidated thickness of the Earth' s crust.

The horizontally homogeneous component $\bar{v}(z)$ of the velocity section, shown in Fig. 1, was found from the hodograph of first arrivals; it is characterized by a smooth increase of velocity with depth.

9. The presence of large-scale inhomogeneities $v_n(x, z)$ is reflected in deviations of the individual smoothed plots of logarithms of amplitudes from the deterministic component $\ln \bar{A}(x)$. This difference, insignificant at distances up to 40 km from the source, increases at distances of 100–120 km to one unit of the natural logarithm. Hence follows the estimate $D \ln A_n(x) : L^3 \approx 10^{-6}$ km^{-3}. The scale of these inhomogeneities is no less than $\sim 0.6\sqrt{R\lambda} \approx 1.2$ km; R is the interval of longitudinal (along the profile) correlation of the high-frequency component of the amplitude plot, determined by the width of the smoothing weighting function and equal to 5 km, $\lambda = 0.8$ km is the predominant wavelength. Inhomogeneities of smaller size are accounted for by the turbidity coefficient.

10. Between the plots $\ln \Delta A$ and ΔT a clear negative correlation is visible; on individual sections of the profile, occupying 1/3 of its total length, the amplitude plots corresponding to different shot points are charac-

are characterized by sharp deviations upward and downward from the mean, up to one unit of the natural logarithm. These intervals have been excluded from consideration, whose aim is to determine the turbidity coefficient.

11. The validity of the adopted approximation of velocity fluctuations by a

locally homogeneous and isotropic field is confirmed by the observance of the theoretical relation (**) between the mean squares of fluctuations of times and of the logarithms of amplitudes.

The turbidity coefficient g km^{-1} was determined by two independent methods: from the reciprocal plots $\ln \delta A(x)$ on profile intervals equidistant from both sources, and from the composite plot $\overline{D} \ln \delta A(r)$. Both methods of determining g are based on a clear analogy with the determination of a velocity section from hodographs of head or refracted waves. The reciprocal plots $\ln \delta A(x)$ made it possible to estimate values of g in several regions of the upper part of the crust down to a depth of 15 km; below, only mean values of g were obtained for the entire investigated segment of the medium. In the upper part these means agree well with the data on the turbidity coefficient obtained from reciprocal systems (see Fig. 1). A decrease in the values of g is characteristic in the depth interval 10–25 km; it is interesting that these depths are in no way noteworthy in the section $\bar{v}(z)$ constructed by us.

The correlation of amplitude and time fluctuations showed that in most cases an increase in amplitude fluctuations corresponds to a decrease in time fluctuations, and conversely. This is explained by the fact that inhomogeneities of different scales are localized at different depths.

12. The mean value of the turbidity coefficient for the crust, corresponding to a frequency of 7 Hz, is about 10^{-3} km^{-1}. This is somewhat less than the values obtained for the crust in the Far East during marine DSS work ([4]). The marine investigations were carried out with less detailed systems, and therefore it was not possible there to differentiate the Earth' s crust by the turbidity coefficient. The seismogeological conditions on our profile are distinguished by very strong inhomogeneity of the upper part of the section. Fluctuations of the amplitudes of the first waves and of their arrival times, caused by these inhomogeneities, sharply limited the possibilities for studying the deterministic component of the section and inhomogeneities of different scales.

13. The model of the medium adopted for interpretation made it possible to explain without contradiction the principal regularities in the relationships between the structure of amplitude plots and hodographs of first waves. In the authors' opinion, such an interpretation for the most part exhausts the useful information on the medium contained in the first waves. Further refinement of the structure of the medium requires the use of information contained in the regions of the wave field following the first arrivals.

Received
27 III 1969

CITED LITERATURE

[1] A. V. Nikolaev, DAN, **177**, No. 5 (1967).
[2] L. A. Chernov, *Wave Propagation in a Medium with Random Inhomogeneities*, Publishing House of the USSR Academy of Sciences, 1958.
[3] A. V. Nikolaev, *Transactions of the Institute of Physics of the Earth*, No. 25 (192) (1962).
[4] I. N. Galkin, A. V. Nikolaev, *Izvestiya of the Academy of Sciences of the USSR, Physics of the Earth*, No. 8 (1968).

Note: Figure translations are in progress. See original paper for figures.