STRUCTURE OF A JET WITH PERIODIC VARIATION OF MOMENTUM

HYDROMECHANICS

1969

SovietRxiv

Abstract

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UDC 532.2

HYDROMECHANICS

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(Presented by Academician Ya. B. Zel' dovich, 17 VII 1968)

Among the various attempts to construct a theory of turbulent motion ([1]), the direct path of solving the nonstationary problem of the motion of a viscous fluid is practically not used. In work ([2]), however, it was noted that, in connection with the development of electronic digital computers, this path, despite the exceptionally great difficulties, does not appear hopeless.

In general form, the scheme of the solution would be conceived as follows. The Navier–Stokes equations for nonstationary fluid motion are integrated with the initial disturbances specified in the form of some, possibly random (probabilistic), spectrum. From the solution obtained, the characteristics of the averaged and pulsational motion are found.

In order to evaluate the prospects of the indicated approach, it is natural to try to simplify the problem as much as possible while retaining its specific character. This purpose is served by the problem considered below, concerning the propagation of a plane nonstationary jet of a viscous fluid, whose solution is carried out within the framework of boundary-layer theory.

Analogously to ([3]), we write the initial system of equations in dimensionless form

$$Sf'(t)\left(u-\xi\frac{\partial u}{\partial \xi}\right)+f(t)\left(u\frac{\partial u}{\partial \xi}+v\frac{\partial u}{\partial \eta}\right)=\frac{1}{\mathrm{Re}}f(t)\frac{\partial^2 u}{\partial \eta^2},$$

$$\frac{\partial u}{\partial \xi}+\frac{\partial v}{\partial \eta}=0, \tag{1}$$

where $f(t)$ is an arbitrary periodic positive function of time ($f>0$; $\xi=x/f(t)$; $\eta=y$; $u=f(t)u(\xi,\eta)$; $v=v(\xi,\eta)$), the dimensionless coordinates and velocity components; S and Re are the Strouhal and Reynolds numbers. Equations (1) are written for flow in a field of constant pressure far from the jet orifice.

We write the boundary conditions in the form

$$v = 0,\ \partial u/\partial\eta = 0 \text{ for } \eta = 0; \qquad u = 0 \text{ for } \eta \to \infty \tag{2}$$

and supplement them (as is usual for a source jet) by the integral relation obtained from equations (1):

$$Sf'(t)\int_0^\infty \left(u - \xi\frac{\partial u}{\partial\xi}\right) d\eta = f(t)\frac{\partial}{\partial\xi}\int_0^\infty u^2\, d\eta. \tag{3}$$

We seek the solution in the form of a series in the small parameter s:

$$u = \sum_0^\infty s^n u_n(\xi,\eta); \qquad v = \sum_0^\infty s^n v_n(\xi,\eta). \tag{4}$$

From the obtained infinite system of differential equations

$$u_0\frac{\partial u_0}{\partial\xi} + v_0\frac{\partial u_0}{\partial\eta} = \frac{1}{R}\frac{\partial^2 u_0}{\partial\eta^2},$$

$$\frac{\partial u_0}{\partial\xi} + \frac{\partial v_0}{\partial\eta} = 0,$$

$$\dots\dots\dots\dots\dots \tag{5}$$

$$f'(t)\left(u_{n-1} - \xi\frac{\partial u_{n-1}}{\partial\xi}\right) + f(t)\sum_{n=0}^{k}\left(u_n\frac{du_{k-n}}{\partial\xi} + v_n\frac{\partial u_{k-n}}{\partial\eta}\right) = \frac{1}{\mathrm{Re}}\frac{\partial^2 u_n}{\partial\eta^2},$$

$$\frac{\partial u_n}{\partial\xi} + \frac{\partial v_n}{\partial\eta} = 0 \tag{6}$$

we shall restrict ourselves to the solution of the equations of the zeroth approximation (5), which contain the basic nonlinearity in the problem (for $n \geqslant 1$ the equations are linear). Equations (5) should be integrated under the boundary conditions (2) and the integral condition

$$\frac{\partial}{\partial\xi}\int_0^\infty u_0^2\, d\eta = 0 \quad \text{or}$$

$$I_{0\xi} = \int_0^\infty \rho u_0^2\, d\eta = \text{const}\,. \tag{7}$$

Fig. 1

In this form the problem formally coincides with the stationary one. Its solution for a self-similar flow is known ([4]), and after returning to the variables x, y, t has the form

$$u(x,y,t) = Af^{1/3}(t)x^{-2/3}\left[1-\mathrm{th}^2\left(By\cdot x^{-2/3}f^{2/3}(t)\right)\right], \tag{8}$$

$$v(x,y,t) = \frac{1}{3}\frac{A}{B}f^{2/3}(t)x^{-2/3}\left\{2By\cdot x^{-2/3}f^{2/3}(t)\left[1-\mathrm{th}^2\left(By\cdot x^{-2/3}f^{2/3}(t)\right)\right]-\mathrm{th}\left(By\cdot x^{-2/3}f^{2/3}(t)\right)\right\}. \tag{9}$$

In formulas (8) and (9) it has been taken into account that, on the basis of equalities (2) and (7),

$$I_x = \int_0^\infty \rho u^2\,dy = I_{0x}f^2(t),$$

where $I_{0x} = \mathrm{const}$.

(10)

Let us pass to the analogue of the averaged turbulent flow. To this end we apply to the obtained solution the usual averaging procedure

$$u = \bar{u} + u', \quad v = \bar{v} + v',$$

$$T = \bar{T} + T', \quad I_x = \bar{I}_x + I'_x,$$

where

$$\bar{u}(x,y) = \frac{1}{\tau}\int_t^{t+\tau} u(x,y,t)\,dt \quad \text{etc.}$$

Fig. 2

Some results of the numerical calculation for $f^2(t) = \cos^{2k} t$, where k is a positive integer, are presented in Figs. 1 and 2. In particular, from

it is seen from the figures that the intensity of the jet decay along the axis increases and the relative velocity profile "fills out" as the exponent k, which reflects the relative intensity of the pulsations, increases (the curves for different values of k in Figs. 1 and 2 correspond to a single value of $\bar{I}_x$)*.

These properties, as is known, are characteristic of turbulent flow. In Fig. 2 there are also plotted the profiles of the turbulent shear stress $\overline{u'v'}$. The profiles of the mean velocity components (as well as of the fluctuating components) and the distribution of $\overline{u'v'}$ have the same form as in turbulent jets [4]. It is curious that the velocity fluctuations $\sqrt{\overline{u'^2}}$ and $\sqrt{\overline{v'^2}}$ obtained in the calculation, as in the experiment, are of the same order of magnitude (despite the absence in the scheme of the equalizing action of pressure fluctuations). The dashed line in Fig. 2 shows the calculated profile of excess temperature (for the physical Prandtl number Pr = 1), corresponding to a value of the turbulent Prandtl number $\mathrm{Pr}_t < 1$, as in the experiments.

Qualitatively analogous results also apply to other forms of the function $f(t)$, corresponding to a periodic change in the jet momentum I_x.

Thus, the results of the simplest numerical experiment qualitatively reflect a number of features of turbulent flow. Apparently, this is connected with the fact that, in the nonstationary equations of the problem, the nonlinearity of the convective terms is retained, while the averaging operation to some extent erases the individual features of the instantaneous motion.

Received14 VII 1968

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* For $\bar{I}_x = \text{const}$, as k increases the ratio $\sqrt{\overline{I^2_{x'}}}/\bar{I}_x$ increases.

Note: Figure translations are in progress. See original paper for figures.