THE POLE TIDE IN THE WORLD OCEAN

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Over the last decade, the most important works have appeared on the 14-month oscillations of the mean level of the World Ocean, known as the pole tide ([1]-[5]). Of particular interest are the studies of I. V. Maksimov ([1]-[3]), in which the phenomenon of the pole tide is considered on a planetary scale on the basis of observational material from 74 stations, which made it possible to establish its principal regularities in the real ocean.

However, some researchers increasingly doubt the reality of the phenomenon under consideration. The basis for these doubts is the circumstance that the amplitudes of the pole tide, calculated by harmonic analysis, differ only slightly in magnitude from the noise level, so that the amplitudes for periods of 13 and 15 months are only slightly smaller than for 14 months ([4]). The results of spectral analysis likewise do not provide convincing evidence, since the peak of the spectrum at the frequency under consideration is insignificant in comparison with the noise level. Thus, the comparatively small magnitude of the amplitude of the pole tide and the fairly broad range of amplitudes of the random oscillations of the series under study do not make it possible to judge reliably the reality of the phenomenon under consideration on the basis of the results of isolating this variation by methods for detecting hidden periodicities. It becomes necessary to obtain such evidence by another methodological route.

Let us consider the basic theoretical premises for the existence of the pole tide. The potential of the centrifugal force caused by the rotation of the Earth, as is known, may be given by the expression ([6]):

$$U = -\frac{1}{2}\omega^2 r^2 \sin^2\theta, \tag{1}$$

where ω is the angular velocity of the Earth' s rotation, r is the mean radius of the Earth, and θ is the co-latitude to 90°.

Owing to the existence of the free nutation of the Earth' s axis of rotation with a period of about 14 months, a corresponding variation of the potential of the centrifugal force arises, called the potential of deformation due to rotation

([4]). Since the displacement of the instantaneous axis of rotation in the body of the Earth leads to a change in latitude, the potential of the deforming force is determined as the differential with respect to latitude of the potential of the centrifugal force:

$$W = dU = -\frac{1}{2}\omega^2 r^2 \cdot 2\sin\theta\cos\theta\, d\theta. \tag{2}$$

Passing to finite increments, let us note that $\Delta\theta$ is the radius vector of the instantaneous pole of rotation, given by the expression: $R = X\cos\lambda + Y\sin\lambda$, where X and Y are the rectangular coordinates of the pole or, what is the same, the projections of R on the meridians 0–180° and 90–270°; λ is longitude. Then

$$W = -\frac{1}{2}\omega^2 r^2 (X\cos\lambda + Y\sin\lambda)\sin 2\theta. \tag{3}$$

Analogous expressions for the potential of the deforming force were given by W. Schweydar ([7]) and W. Munk ([4]).

The deforming force, acting on the oceanic shell of the Earth in the manner of tidal forces, should lead to the formation in the ocean of a "tide" with the corresponding 14-month period. The height of the tide is determined by the formula of the static theory:

$$H = \frac{W}{g}(1 + k - h), \tag{4}$$

where $(1 + k - h)$ is a factor characterizing the elastic properties of the Earth. Substituting into (3) the expression for the potential of the force, we obtain:

$$H = -\frac{(1 + k - h)}{2g}\,\omega^2 r^2 (X\cos\lambda + Y\sin\lambda)\sin 2\theta. \tag{5}$$

Relations (2)–(5) make it possible to draw several conclusions about the character of the polar tide in the ocean:

1. From equation (3) it follows that the magnitude of the potential of the deforming force is opposite in sign to the projection of the radius vector of the instantaneous pole of the Earth' s rotation on the 0–180° meridian. In other words, when the radius vector crosses the Greenwich meridian in a given region of the terrestrial globe, there is a minimum of the force potential; further, judging from expression (3), the magnitude of the deforming force changes correspondingly with longitude. Therefore, assuming that at some definite instant the projection X is maximal, i.e. the radius vector intersects the Greenwich meridian, and that its phase φ_X is correspondingly 90° *, we obtain the values of the phases of the potential of the deforming

force for different longitudes of the northern and southern hemispheres (in the southern hemisphere the picture, evidently, will be reversed) (Table 1).

Table 1

Variation with longitude of the phase value of the potential of the deforming force φ_W

$(\varphi_X = 90^\circ)$

		0°	30°	60°	90°	120°	150°
φ_W	Longitudes	0°	30°	60°	90°	120°	150°
φ_W	Northern hemisphere	270	240	210	180	150	120
φ_W	Southern hemisphere	90	60	30	360	330	300
		180°	150°	120°	90°	60°	30°
φ_W	Longitudes	180°	150°	120°	90°	60°	30°
φ_W	Northern hemisphere	90	60	30	360	330	300
φ_W	Southern hemisphere	270	240	210	180	150	120

Expression (5) shows that the variation with longitude of the phase of the polar tide must have an analogous character.

2. The change in the magnitude of the potential of the deforming force with latitude can be judged from the form of expressions (2), (5), which include a term containing the complement to the latitude θ. Accordingly, the amplitudes of the polar tide should also vary with latitude, reaching maximum values at 45° north and south latitude and being equal to zero at the equator and at the poles. We computed, from relation (5), the amplitudes of the static polar tide with the following initial data: $\omega = 7.29 \cdot 10^{-5}$ sec^{-1}, $r = 6378$ km, $(1 + k - h) = 0.69$, $g = 980$ cm/sec^2, $X = 0.22$ (according to the results of a harmonic analysis of the coordinates of the pole from 1947 to 1960), $\lambda = 0^\circ$. These calculations gave the following results:

Fig. 1. Comparison of the changes with latitude in the amplitudes of the static pole tide (d) with the latitudinal distribution of the amplitudes of 14-month fluctuations of the mean level of various zones of the World Ocean at $\lambda_{\text{cp}} = 02°00'E$ (a), $132°00'E$ (b), $128°00'W$ (c), $69°00'W$ (d).

Figure 1: Fig. 1. Comparison of the changes with latitude in the amplitudes of the static pole tide (d) with the latitudinal distribution of the amplitudes of 14-month fluctuations of the mean level of various zones of the World Ocean at $\lambda_{\text{cp}} = 02°00'E$ (a), $132°00'E$ (b), $128°00'W$ (c), $69°00'W$ (d).

Latitude	90°	80°	70°	60°	50°	45°	40°	30°	20°	10°	0°
Amplitudes, mm	0.0	2.8	5.2	7.0	8.0	8.1	8.0	7.0	5.2	2.8	0.0

* Here and below, the phase φ is the phase of a sinusoidal wave of the form $A \sin\left(\frac{2\pi}{T}t + \varphi\right)$, whence it follows that at $\varphi = 90°$ the quantity being analyzed at the initial instant of time $t = 0$ has a maximum value.

Judging from the data obtained, the amplitude of the static pole tide does not exceed 8 mm.

Thus, the theoretical conception of the nature of the pole tide in the ocean is based on the proposition that the phase of the tide varies with longitude and on the law of the latitudinal distribution of amplitudes.

Let us turn to the results of the analysis of fluctuations of the level of the World Ocean. The amplitudes and phases of the 14-month fluctuations of mean level were computed on an electronic computer by the method of harmonic analysis of 14-month series of monthly mean level heights at 275 stations of the World Ocean for the period from 1947 to 1960 or from 1948 to 1961, and in some cases for 1900–1913 ([8]), i.e., for the years of the greatest range of the free oscillations of the pole, when the pole tide should naturally appear especially clearly. Then all the values of the initial phases of the level oscillations were reduced to a single instant of time, taken to be the instant when the radius vector crossed the Greenwich meridian. It was indicated above that if the radius vector crosses the Greenwich meridian at an instant coinciding with the beginning of the analyzed series of values of the coordinate X, then the initial phase of the 14-month oscillation X, according to the results of harmonic analysis, will be equal to 90°. The deviation of the phase from 90°, i.e. $\Delta\varphi = 90° - \varphi_X$, is, in essence, the time interval between the instant when the radius vector crosses the Greenwich meridian and the beginning of the analyzed series.

Fig. 1. Comparison of the changes with latitude in the amplitudes of the static pole tide (d) with the latitudinal distribution of the amplitudes of 14-month fluctuations of the mean level of various zones of the World Ocean at $\lambda_{\text{cp}} = 02°00'E$ (a), $132°00'E$ (b), $128°00'W$ (c), $69°00'W$ (d).

Consequently, by adding the values of the obtained initial phases of the level oscillations to the quantities of the deviations ($90^\circ - \varphi_X$), where the values of φ_X are computed by harmonic analysis of the series of the coordinate X corresponding in time interval to the series of level data, we reduce all initial phases to the instant when the radius vector crosses the Greenwich meridian. This reduction was carried out according to the formula:

$$\varphi_{\text{Gr}} = \varphi_{\text{n}} + 90^\circ - \varphi_X,$$

where φ_{Gr} is the phase of the level oscillation reduced to the instant when the radius vector crosses the Greenwich meridian; φ_{n} is the initial phase of the 14-month level oscillation, computed by harmonic analysis; φ_X is the phase of the 14-month oscillation of the coordinate X.

The results of the analysis of fluctuations of the mean level of the World Ocean are presented in Table 2 and in Fig. 1.

Considering these results in comparison with the theoretical conclusions set forth above, we arrive at the following principal proofs of the existence of the pole tide in the World Ocean:

1. The wave of the pole tide, being a forced wave, should lag only slightly (owing to friction) in phase behind the disturbing–

Table 2

Comparison of the phase values of 14-month sea-level oscillations $\varphi_{\text{Tr}}{}^*$ with the phases of the potential of the deforming force φ_W

Regions of the World Ocean	Mean longitude	φ_{Tr}, deg	φ_W, deg
Northern Hemisphere			
Atlantic Ocean, eastern coast (35 stations)	$03^\circ 00' E$	267	267
North Sea (12 stations)	$05^\circ 00' E$	271	265
Mediterranean Sea (10 stations)	$10^\circ 00' E$	276	260
Baltic Sea (20 stations)	$21^\circ 30' E$	248	248
Barents Sea (4 stations)	$35^\circ 00' E$	220	235

Regions of the World Ocean	Mean longitude	φ_{Tr}, deg	φ_W, deg
Kara Sea (5 stations)	$72°30'E$	183	198
Laptev Sea (5 stations)	$124°00'E$	155	146
Pacific Ocean, eastern coast (14 stations)	$137°00'E$	136	133
Chukchi and Bering Seas (6 stations)	$175°00'E$	82	85
Pacific Ocean, western coast (16 stations)	$127°30'W$	35	37
Atlantic Ocean, western coast (17 stations)	$69°00'W$	336	339
Southern Hemisphere			
Atlantic Ocean, western coast (7 stations)	$60°00'W$	170	150
Pacific Ocean, western coast (5 stations)	$71°00'W$	165	161
Pacific Ocean, eastern coast (4 stations)	$149°00'E$	326	301

* Since the phase of nutational sea-level oscillations also varies with latitude, differing in the Southern Hemisphere by 180°, when averaging over longitudes the phase values were taken for stations lying between the parallels 35° and 70°, respectively, in the Northern and Southern Hemispheres.

of the force. Consequently, the phases of the 14-month oscillations of mean sea level must correspond, or almost correspond, to the phases of the potential of the deforming force in different longitudinal zones of the World Ocean. This proposition is fully confirmed by the data of Table 2.

2. The character of the distribution of the amplitudes of the 14-month sea-level oscillations by latitude in four zones of the World Ocean, considerably

distant from one another, in comparison with the curve of the distribution of amplitudes of the static polar tide, presented in Fig. 1, corresponds to the law of the latitudinal distribution of amplitudes, which is another important confirmation of the nutational nature of the sea-level oscillations under consideration.

Thus, according to the propositions presented, the reality of the existence of the polar tide, in our opinion, is beyond doubt, and the role of this phenomenon in the formation of circulation processes in the World Ocean becomes an object of study.

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Note: Figure translations are in progress. See original paper for figures.