ON AN INVERSE PROBLEM FOR THE TELEGRAPH EQUATION

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Abstract

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MATHEMATICS

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ON AN INVERSE PROBLEM FOR THE TELEGRAPH EQUATION

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This paper is devoted to finding the coefficient for the telegraph equation.

Consider the equation

$$u_{xy} = au \tag{1}$$

with data on the characteristics

$$u(x,y,\lambda)\big|_{x=0} = g(y-\lambda), \qquad u(x,y,\lambda)\big|_{y=0} = 0, \tag{2}$$

$$u(x,y,\lambda)\big|_{y=c} = \varphi(x,\lambda), \tag{3}$$

where $u(x,y,\lambda)$ is a family of solutions of (1), depending on the parameter λ $(0<\lambda<c)$; $a=a(x,y)$ is the coefficient of equation (1); $g(y-\lambda)$ is the Dirac function; $\varphi(x,\lambda)$ is a function having a continuous mixed second derivative with respect to x and λ in the rectangle $Q=[0,h;0,c]$. The rectangle Q is bounded by the coordinate axes and the straight lines $x=h,\ y=c$; h and c are positive constants.

Let $a(x,y)$ be an unknown continuous function in the rectangle Q, and let

$$\sup_{0\le x\le h,\ 0\le y\le c} |a(x,y)| = M,$$

where M is some positive number.

Let c and h satisfy the condition

$$M^* \operatorname{ch} S < 1, \tag{4}$$

where S and M^* are certain positive numbers, which will be discussed below.

Multidimensional inverse problems in a linearized formulation were considered in work (1).

In this paper the problem is considered in the following formulation: for a given function $\varphi(x,\lambda)$ in the rectangle Q, it is required to find the coefficient $a(x,y)$ of equation (1) in this rectangle.

The solution of equation (1) satisfying the boundary conditions (2) is equivalent to the solution of the integral equation:

$$u(x,y,\lambda) = g(y-\lambda) + \int_0^y \int_0^x d(\xi,\eta)\, u(\xi,\eta,\lambda)\, d\xi\, d\eta. \tag{5}$$

Equation (5) is a Volterra integral equation of the second kind with respect to $u(x,y,\lambda)$.

From (5), applying the method of successive approximations, we obtain

$$u(x,y,\lambda) = g(y-\lambda) + \sum_{n=1}^{\infty} \int_0^y \int_0^x g(t-\lambda)\, a_n(x,y,\tau,t)\, d\tau\, dt, \tag{6}$$

where

$$a_n(x,y,\tau,t) = \int_t^y \int_\tau^x a(\xi,\eta) a_{n-1}(\xi,\eta,\tau,t)\, d\xi\, d\eta$$

is the n-th iterated kernel for $a(x,y)$. In view of the boundedness of $a(x,y)$ in the rectangle Q, the series in (6) will converge absolutely and uniformly in this rectangle. Therefore, interchanging the order of summation and integration in (6), we obtain

$$u(x,y,\lambda) = g(y-\lambda) + \int_0^y \int_0^x g(t-\lambda) R(x,y,\tau,t)\, d\tau\, dt, \tag{7}$$

where

$$R(x,y,\tau,t) = \sum_{n=1}^{\infty} a_n(x,y,\tau,t), \qquad a_1 = a(\tau,t).$$

From (7), by virtue of condition (3), we obtain

$$\varphi(x,\lambda) = g(c-\lambda) + \int_0^c \int_0^x g(t-\lambda) R(x,c,\tau,t)\, d\tau\, dt. \tag{8}$$

Differentiating relation (8) with respect to x, and then with respect to λ, we obtain

$$-\varphi''_{x\lambda}(x,\lambda) = a(x,\lambda) + \int_{\lambda}^{c}\int_{0}^{x} a(x,\eta)R(x,\eta,\tau,\lambda)\,d\tau\,d\eta. \tag{9}$$

Relation (9) can be written in the form

$$a(x,\lambda) = Aa, \tag{10}$$

where

$$Aa = -\varphi''_{x\lambda}(x,\lambda) - \int_{\lambda}^{c}\int_{0}^{x} a(x,\eta)R(x,\eta,\tau,\lambda)\,d\tau\,d\eta \tag{11}$$

is a nonlinear operator.

We now show that this operator is a contraction operator in the space $C[0,h;0,c]$. Take two arbitrary elements $a(x,y)$ and $b(x,y)$ from the space $C[0,h;0,c]$,

$$\rho(a,b) = \max_{x,y\in C[0,h;0,c]} |a-b|.$$

Let for $b(x,y)$ in the rectangle Q the condition

$$\sup_{0\le x\le h,\ 0\le y\le c} |b(x,y)| = M_1$$

be satisfied.

Denote by M^* the larger of the numbers M and M_1. Using relation (11), we write $\rho(Aa, Ab)$:

$$\rho(Aa,Ab) = \|Aa - Ab\| =$$

$$= \left\|\int_{\lambda}^{c}\int_{0}^{x} [b(x,\eta)R^*(x,\eta,\tau,\lambda) - a(x,\eta)R(x,\eta,\tau,\lambda)]\,d\tau\,d\eta\right\|, \tag{12}$$

where

$$R^*(x,\eta,\tau,\lambda) = \sum_{n=1}^{\infty} b_n(x,\eta,\tau,\lambda), \qquad b_1 = b(\tau,\lambda).$$

From (12) we obtain

$$\rho(Aa, Ab) \leqslant \rho(a, b) M^* hcS, \tag{13}$$

where S is the sum of the series

$$\sum_{n=1}^{\infty} \frac{(n+1)\,|M^* hc|^{n-1}}{(n-1)!\,(n-1)!}.$$

From (13), in view of condition (4), we see that the operator A is a contraction operator. Therefore, on the basis of the contraction mapping principle, from the functional equation (10) we determine the unique coefficient $a(x, y)$ in the rectangle Q.

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CITED LITERATURE

1. M. M. Lavrent' ev, V. G. Romanov, DAN, **171**, No. 6 (1966).

2. S. Elubaev, Vestn. AN KazSSR, 1969.

Note: Figure translations are in progress. See original paper for figures.

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