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Abstract

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ON THE THEORY OF THE EFFECT OF RADIATION ON THE MOTION OF ELECTRONS IN ACCELERATORS

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1. The question of the effect of radiation on the motion of electrons in accelerators and, in particular, the question of the relationship between the classical and quantum approaches to solving this problem has been widely discussed in the literature and has caused a lengthy controversy (for a review of the literature, see ([1,2])). Recently the authors proposed a method for the quantum treatment of the radiation process of high-energy particles in an arbitrary magnetic field ([3]). Within the framework of this method one can simply and in a unified way consider problems concerning the effect of radiation on the motion of particles in a magnetic field in general.

It should be borne in mind that quantum effects in magnetic bremsstrahlung are of two types: quantization of the electron' s motion itself in the magnetic field (taking it into account leads to corrections* $\sim \hbar\omega_0/E = \hbar/Rm\gamma$ ($\gamma = E/m$); for $E = 1$ GeV and $R = 200$ cm, $\hbar/Rm\gamma \sim 10^{-16}$) and quantum recoil in the emission of a photon (the magnitude of the corresponding effects $\sim \hbar\omega_0\gamma^3/E$). Therefore, for high-energy particles ($\gamma \gg 1$) one may neglect the quantization of orbits in an accelerator (i.e., the motion of electrons is quasiclassical) and take into account only the recoil in photon emission. In essence, the problem of radiation in a magnetic field has been solved only in this approximation. The properties of radiation depend essentially on the parameter $\chi = \hbar\omega_0\gamma^3/E$; the region $\chi \ll 1$ is almost classical (only it will be considered by us below), while the region $\chi \gtrsim 1$ is essentially quantum.

The probability of emitting a photon per unit time at the instant t is given by the expression ([3])**

$$dW = \alpha \frac{d^3k}{(2\pi)^2\omega} \int_{-\infty}^{\infty} \varphi(\tau)\, d\tau, \tag{1}$$

$$\varphi(\tau) = \left[\frac{1+u}{\gamma^2} + \frac{1}{2}\left(1 + u + \frac{u^2}{2}\right)\dot{\mathbf{v}}^2\tau^2\right] e^{-i\frac{u\tau E}{\hbar}\left(1 - \mathbf{nv} + \frac{\dot{\mathbf{v}}^2\tau^2}{24}\right)},$$

where $\alpha = e^2/4\pi\hbar = 1/137$; $u = \hbar\omega(E - \hbar\omega)$; $\mathbf{n} = \mathbf{k}/\omega$ is the direction of emission of the photon; ω is the photon frequency; $\mathbf{v}, \dot{\mathbf{v}}$ are the velocity and acceleration of the electron. This expression is valid for any χ; in the transition to the classical limit, $u \sim \hbar\omega/E \ll 1$. Formula (1) contains a time description of the radiation process, with the integrand $\varphi(\tau)$ giving the dependence on the time τ of the probability density for photon emission. This function is rapidly oscillating, so that the main contribution to integral (1) is made by $\tau \lesssim 1/|\dot{\mathbf{v}}|\gamma \sim 1/\omega_0\gamma$, which determines the formation time of the photon. Then, in the laboratory system, owing to the Doppler effect, the characteristic frequencies of the emitted photons turn out to be

$$\omega \sim \frac{1}{\tau}\gamma^2 \sim \omega_0\gamma^3.$$

Here it was assumed that the change of the magnetic field over the formation length

* A system of units with $c = 1$ is used.

** Here and below we shall retain only the leading terms of the expansion in $1/\gamma$.

the photon can be neglected. Upon integrating $\varphi(\tau)$ together with an arbitrary smooth function $g(\tau)$, we have

$$\int g(\tau)\varphi(\tau)\, d\tau \simeq g(0)\left(1 + \text{const}\,\frac{q}{\gamma}\right)\int \varphi(\tau)\, d\tau,$$

where $\Omega_g = \omega_0 q = |g'(0)/g(0)|$ is the characteristic frequency of variation of the function $g(\tau)$. Hence it is seen that, to within terms $\sim q/\gamma$, the radiation probability density has the properties of a δ-function.

The mean number of photons emitted per unit time is determined by $W = \int dW$, so that the mean time between photon emissions is $1/W \sim 1/\alpha\gamma\omega_0$. Thus, the time pattern of radiation consists of bursts of duration $\sim 1/\omega_0\gamma$, separated by a mean interval $\sim 1/\alpha\gamma\omega_0$, and the possibility of representing it as a sequence of separate bursts is connected with the smallness of the interaction constant α. Above we considered the emission of a photon by a given (“classical”) current. As is known (see, for example, ([4])), the probability of photon emission by a given current in quantum electrodynamics is described by a Poisson distribution, and the process of successive photon emission (i.e., the distribution in time of the bursts indicated above) is statistically independent.

2. Let us find, within the framework of the method ([3]), the change in the mean value of a function of the momentum operator $F(p_\mu(t))$ for states before and after emission, $|t_0\rangle$ and $|t\rangle = U(t, t_0)|t_0\rangle$, where $U(t, t_0) =$

$1-i\int_{t_0}^{t} dt_1 H_{\text{int}}(t_1)$ ..., $H_{\text{int}}(t_1)$ is the Hamiltonian of the interaction of the particle in the magnetic field with the radiation field ([3]),

$$\langle t|F(\hat{p}_\mu(t))|t\rangle - \langle t_0|F(\hat{p}_\mu(t_0))|t_0\rangle = \langle t_0|U^+F(\hat{p}_\mu(t))U - F(\hat{p}_\mu(t_0))|t_0\rangle$$

$$= \langle t_0|U^+[F(\hat{p}_\mu(t)),U] + F(\hat{p}_\mu(t)) - F(\hat{p}_\mu(t_0))|t_0\rangle. \tag{2}$$

To calculate the commutator entering (2), we take into account that: 1) only the term $e^{-ikr(t_1)}$ in $H_{\text{int}}(t_1)$ does not commute with the operator $F(\hat{p}_\mu(t))$; 2) the emission process occurs over a very short time, so that the interval $t - t_0$ may be chosen small and, in the commutator, an expansion in the time difference may be carried out, retaining only the leading term of the expansion; 3) $\langle t_0|[F(\hat{p}_\mu),U]|t_0\rangle \simeq 0$.

Then we obtain

$$\langle t_0|U^+[F(\hat{p}_\mu),U]|t_0\rangle =$$

$$= \left\langle t_0 \left| \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\, H^+_{\text{int}}(t_2)H_{\text{int}}(t_1)\,[F(\hat{p}_\mu - \hbar k_\mu) - F(\hat{p}_\mu)] \right| t_0 \right\rangle. \tag{3}$$

Carrying out the further calculations as in ([3]), and replacing the operators in the square brackets by their classical values, we obtain

$$\frac{dF(p_\mu)}{dt} = \int dW\,[F(p_\mu(t) - \hbar k_\mu) - F(p_\mu(t))] + \left(\frac{dF}{dt}\right)_i, \tag{4}$$

where the last term describes the change of the function F not connected with radiation. Equation (4) is a quasiclassical generalization of the radiation reaction. The domain of applicability of formula (4) is the same as that of formula (1). With the aid of equation (4), any problem on the effect of radiation on the motion of particles of high energy ($\gamma \gg 1$) in a magnetic field can be solved.

3. We shall apply (1), (4) to the dynamics of electrons in accelerators; moreover, we shall be interested in characteristics averaged over the oscillations in the case when they vary slowly with time.

Radiation has a twofold effect on the motion of electrons in an accelerator. On the one hand, for an electron executing small transverse oscillations, radiation directed mainly within an angle $\sim 1/\gamma$

around the direction of the velocity, carries away the total momentum; and since only the longitudinal momentum is replenished, the transverse oscillations are damped. In this case the radial (ρ) and phase (X) potential wells in which the particles move in the accelerator depend on the energy, so that a change in energy may also lead to damping or excitation of the ρ- and X-oscillations*. On the other hand, owing to the discrete nature of the process of energy emission (and, consequently, the equilibrium radius R changes by jumps), the collection of such kicks leads to statistical excitation of the ρ- and X-oscillations (the so-called quantum excitation). Excitation of the vertical z-oscillations occurs due to the small transverse recoil of the electron during emission; for the ρ- and X-oscillations the recoil effect may be neglected. The phenomenon of quantum excitation was first indicated by A. A. Sokolov and I. M. Ternov (see ([2])).

Let us turn to a concrete consideration of the effect of radiation on the oscillations. For simplicity we shall assume that the oscillations in the absence of radiation are independent and that the total energy of the electron does not change on the average. In this problem it is convenient to use the Hamiltonian formalism. Substituting the energy of the oscillatory motion

$$\mathcal{H}_j = p_j^2/2E + U_j(q_j) = T_j + U_j \tag{5}$$

into (4), we obtain

$$\frac{d\mathcal{H}_j}{dt} = \frac{1}{2E}\int \left(-2p_j\hbar k_j + \hbar^2 k_j^2\right) dW +$$

$$+ \int \left[U_j(q_j(E-\hbar\omega)) - U_j(q_j(E))\right] dW + \frac{dU_j}{dq_j}\frac{dq_j}{dE}\left(\frac{dE}{dt}\right)_i. \tag{6}$$

Keeping the leading terms of the expansion up to second order in $(\hbar\omega/E)$ and taking into account that $\dot{E} = -I + \left(\frac{dE}{dt}\right)_i$, where $I = \int \hbar\omega\, dW$ is the radiation intensity, and that

$$\int \hbar k_j\, dW \simeq \left(v_j + \text{const}\,\frac{1}{\gamma^2}\right) I,$$

we obtain

$$\frac{d\mathcal{H}_j}{dt} = -2T_j\frac{I}{E} + \frac{1}{2E}\int \hbar^2 k_j^2\, dW + \frac{dU_j}{dq_j}\frac{dq_j}{dE}\dot{E} + \frac{1}{2}\frac{d^2U_j}{dq_j^2}\left(\frac{dq_j}{dE}\right)^2 \int \hbar^2\omega^2\, dW. \tag{7}$$

Taking into account that $\dot{E} \simeq \frac{d\dot{E}}{dq_j}q_j$ ($\dot{E}_{q=0} = 0$), carrying out the averaging over time under the assumption that $I/E \ll \omega_j$, and using the virial theorem

$$\frac{dU_j}{dq_j} q_j = 2\overline{T}_j,$$

we obtain

$$\frac{d\mathcal{H}_j}{dt} = \left(\frac{dq_j}{dE} \frac{d\dot{E}}{dq_j} - \frac{I}{E} \right) 2\overline{T}_j(\mathcal{H}_j) + \frac{\hbar^2}{2E} \int k_j^2 \, dW + \frac{1}{2} \overline{\frac{d^2 U_j}{dq_j^2} \left(\frac{dq_j}{dE} \right)^2} \int \hbar^2 \omega^2 \, dW. \tag{8}$$

The formula obtained is applicable to any potential. For oscillator potentials $2\overline{T}_j = E_j$, we have

$$\frac{dE_\rho}{dt} = \left(-\frac{dR}{dE} \frac{d\dot{E}}{dr} - \frac{I}{E} \right) E_\rho + \frac{55\alpha}{48\sqrt{3}} \hbar\omega_\rho \left(\frac{\omega_\rho}{\omega_0} \right) \left(\frac{d\ln R}{d\ln E} \right)^2 \frac{\omega_0^2}{m} \gamma^6; \tag{9}$$

$$\frac{dE_X}{dt} = \frac{dR}{dE} \left(\frac{d\dot{E}}{dr} + \frac{d\dot{E}}{dE} \frac{dE}{dR} \right) E_X + \frac{55\alpha}{48\sqrt{3}} \hbar\omega_X \left(\frac{\omega_X}{\omega_0} \right) \left(\frac{d\ln R}{d\ln E} \right)^2 \frac{\omega_0^2}{m} \gamma^6; \tag{10}$$

$$\frac{dE_z}{dt} = -\frac{I}{E} E_z + \frac{13\alpha}{48\sqrt{3}} \hbar\omega_0 \frac{\omega_0^2}{m} \gamma^4. \tag{11}$$

* We shall again neglect the quantization of the oscillations, since taking it into account gives negligibly small corrections $\sim \hbar\omega_j / E_j$, where ω_j and E_j are the frequency and energy of the corresponding oscillatory motion.

Here it has been used that $\rho = r - R$, $X = R - R_0$ (R_0 is the radius of the equilibrium orbit), $\dot{E} = \dot{E}(r, E(R))$; for z-oscillations the potential well does not depend on the energy. The last term in each of equations (9)–(11) gives the quantum buildup; the remaining terms are classical.

Taking into account that $d\dot{E}/dE = -dI/dE$, for the sum of the decrements we have

$$\Gamma_\rho + \Gamma_X = \frac{I}{E} \left(1 + \frac{d\ln I}{d\ln E} \right), \tag{12}$$

which is the well-known assertion that the sum of the decrements is independent of the particular properties of the system. The assumption made above about the independence of the oscillations is not fundamental (although it simplifies the calculation), since it is not difficult to obtain an analogue of formula (8) in the general case.

4. The relationship between the quantum and classical terms on the right-hand side of formulas (8)–(11) at one time provoked a lengthy controversy. On the one hand, attempts were made to obtain the terms with classical damping from a quantum calculation using approximate wave functions in a weakly inhomogeneous field (Gutfrod; Sokolov, Ternov [2]). It should be noted that the method used in these works is essentially quasiclassical and cannot claim greater rigor than that used in the present article. Similar work using perturbation theory, also for nonrelativistic particles, was carried out by Yu. F. Orlov and S. A. Kheifets [5]. On the other hand, the terms giving the quantum buildup were obtained by A. A. Kolomenskii and A. N. Lebedev under the assumption of δ-functional random forces (naturally, only to the lowest order in $\hbar$). Such an approach is adequate, and the assumptions made follow from quantum electrodynamics, after which the problem of a quantum-mechanical justification is naturally removed.

Proposals have repeatedly been made to change the character of magnetic bremsstrahlung radiation in accelerators by means of quantum effects (quantization of orbits, prohibitions on transitions in the corresponding potential wells, etc.). We wish to note here that the radiation of high-energy particles in a magnetic field at arbitrary χ has a quasiclassical character and is determined only by the magnitude of the acceleration $|\mathbf{v}|$ (in the case when the field does not change over the photon formation length), whereas the effects of quantization of the motion have an insignificantly small influence on the radiation ($\sim \hbar\omega_j/E$), so that attempts at quantum suppression of magnetic bremsstrahlung radiation cannot count on success.

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Note: Figure translations are in progress. See original paper for figures.