DOUBLE INSTABILITY OF HOMOGENEOUS PRECESSION

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Abstract

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DOUBLE INSTABILITY OF HOMOGENEOUS PRECESSION

IN FERROMAGNETIC RESONANCE

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The instability of homogeneous precession of the magnetization is caused by the dependence of the resonance frequency on the amplitude of the oscillations. The source of such instability may be, for example, shape anisotropy ([1,2]). At the same time, instability of this kind was also observed in ferromagnetic resonance on spherical specimens ([3]). An explanation of this effect, as shown in ([4–7]), is possible only when crystallographic anisotropy is taken into account. In the preceding works, however, either the precession angle was assumed small, or the shape anisotropy and crystallographic anisotropy were not taken into account simultaneously. The present work is an attempt to overcome these shortcomings.

The motion of the magnetization vector $\mathbf{M}$ in a small homogeneous specimen is described by the Landau-Lifshitz equation:

$$\frac{d\mathbf{M}}{dt} = -\gamma[\mathbf{M}\mathbf{H}] + \frac{\alpha}{M_0}\left[\mathbf{M}\frac{d\mathbf{M}}{dt}\right]. \tag{1}$$

Here γ is the absolute value of the gyromagnetic ratio, $\alpha < 1$ is the phenomenological damping constant, and M_0 is the saturation magnetization.

Most ferrites have a cubic lattice. Let us direct the axes of the coordinate system along the crystallographic axes [100], [010], and [001], respectively. Then the effective magnetic field $\mathbf{H}$—the sum of the static field, the demagnetizing field, the anisotropy field, and the alternating high-frequency field ν—is

$$H_i = H_{0i} - 4\pi N_i M_i + 2\left(K_1/M_0^4\right)M_i^3 + h_i(\nu t) \qquad (i = x, y, z),$$

where $\sum N_i = 1$, K_1 is the first anisotropy constant, and the quantum-mechanical exchange term has been neglected because of its smallness.

Let $\mathbf{H}_0$ be directed along the z-axis, and let $\mathbf{h}(\nu t)$ lie in the xy-plane:

$$h_x(\nu t) = h_x \cos\nu t, \qquad h_y(\nu t) = h_y \sin\nu t, \qquad h_z(\nu t) = 0.$$

With this notation one can obtain all possible polarizations of the microwave field. Introducing the notation

$$\mathbf{m} = \mathbf{M}/M_0, \qquad \omega = \gamma H_0, \qquad \chi = \gamma K_1/M_0, \qquad \xi_j = \gamma 4\pi M_0 (N_z - N_j), \qquad \eta_j = \gamma h_j \; (j = x, y),$$

we obtain from (1), for the transverse components of the magnetization:

$$\dot{m}_x = -m_y \left[\omega - \xi_y m_z + 2\chi m_z \left(m_z^2 - m_y^2\right)\right] + m_z \eta_y \sin\nu t + \alpha \left(m_y \dot{m}_z - m_z \dot{m}_y\right)$$

$$\dot{m}_y = m_x \left[\omega - \xi_x m_z + 2\chi m_z \left(m_z^2 - m_x^2\right)\right] - m_z \eta_x \cos\nu t + \alpha \left(m_z \dot{m}_x - m_x \dot{m}_z\right). \tag{2}$$

To solve the system of nonlinear equations (2) by the Krylov-Bogolyubov asymptotic method, it is necessary to introduce into the system a dimensionless small parameter ε and to arrange the terms according to ε.

Usually χ, ξ_j, η_j are smaller than ν, and $\omega \simeq \nu$. We shall therefore regard χ/ν, ξ_j/ν, η_j/ν, and α as quantities of order ε. Then $\dot{m}_z$ is also a quantity of order ε. Eliminating m_z from (2) by means of the identity $1 = m_x^2 + m_y^2 + m_z^2$ and restrict-

retaining terms of first order of smallness, we find:

$$\begin{aligned} \dot{m}_x + \omega m_y &= \varepsilon f(m_x, m_y, \nu t) + \varepsilon^2 \dots, \\ \dot{m}_y - \omega m_x &= \varepsilon g(m_x, m_y, \nu t) + \varepsilon^2 \dots, \end{aligned} \tag{3}$$

where

$$\varepsilon f = \sqrt{1 - (m_x^2 + m_y^2)} \left[(\xi_y - 2\chi) m_y + 2\chi m_y (m_x^2 + 2m_y^2) + \eta_y \sin\nu t - \alpha\omega m_x\right],$$

$$\varepsilon g = -\sqrt{1 - (m_x^2 + m_y^2)} \left[(\xi_x - 2\chi) m_x + 2\chi m_x (2m_x^2 + m_y^2) + \eta_x \cos\nu t + \alpha\omega m_y\right].$$

According to the Krylov–Bogoliubov method [8], the solution of system (3) near resonance will, in the first approximation, be

Figure 1. Curves of the dependence of a^2 on frequency (for $K_1 < 0$). 1—$\eta < \eta_{\mathrm{cr1}}$; 2—$\eta = \eta_{\mathrm{cr1}}$; 3—$\eta_{\mathrm{cr1}} < \eta < \eta_{\mathrm{cr2}}$; 4—$\eta = \eta_{\mathrm{cr2}}$; 5—$\eta > \eta_{\mathrm{cr2}}$.

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$$m_x = a\cos\psi, \qquad m_y = a\sin\psi, \qquad \psi = \nu t + \rho,$$

where the amplitude a and the phase difference ρ depend on time and are determined by the system of equations:

$$\begin{aligned} \frac{da}{dt} &= -\sqrt{1-a^2}(\alpha\omega a + \eta\sin\rho), \\ \frac{d\rho}{dt} &= \omega - \nu(a) - \frac{1}{a}\sqrt{1-a^2}\eta\cos\rho. \end{aligned} \tag{4}$$

Here

$$\nu(a) = \nu + \sqrt{1-a^2}\left({}^7/_2\chi a^2 + \xi - 2\chi\right); \qquad \xi = {}^1/_2(\xi_x + \xi_y); \quad \eta = {}^1/_2(\eta_x + \eta_y).$$

In the stationary regime $\dot{a} = 0$; $\dot{\rho} = 0$. Then, eliminating ρ from (4), we obtain in the first approximation the following relation between the amplitude of the forced oscillations and the frequency (the equation of the resonance curve):

$$\omega = \nu + \sqrt{1-a^2}\left\{{}^7/_2\chi a^2 + \xi - 2\chi \pm \sqrt{\eta^2/a^2 - \alpha^2\nu^2(a)}\right\}. \tag{5}$$

Equation (5) determines the shift of the resonance frequency for arbitrary precession angles, since $a^2 = m_x^2 + m_y^2 = \sin^2\theta$, where θ is the angle between $\mathbf{H}_0$ and $\mathbf{M}$.

In the case of small a^2, one can expand $\sqrt{1-a^2}$. Restricting ourselves to the first terms of the expansion, we obtain the results of works [4–7]. In this case the zero-order term, independent of a^2, is equal to $\nu + \xi - 2\chi$; this is the well-known expression for the resonance frequency with account taken of the anisotropy and the shape of the specimen [9].

Fig. 1. Curves of the dependence of a^2 on frequency (for $K_1 < 0$). 1—$\eta < \eta_{\mathrm{cr1}}$; 2—$\eta = \eta_{\mathrm{cr1}}$; 3—$\eta_{\mathrm{cr1}} < \eta < \eta_{\mathrm{cr2}}$; 4—$\eta = \eta_{\mathrm{cr2}}$; 5—$\eta > \eta_{\mathrm{cr2}}$.

Let us construct, with the aid of (5), the resonance curves (Fig. 1) for various values of the parameter η (the amplitude of the microwave field). For small η the resonance curve is symmetric with respect to $\omega = \nu + \xi - 2\chi$; this is the region of the linear theory. As η increases, the resonance curve is deformed. The direction of the bending, and consequently also the sign of the detuning of the resonance frequency, depends on the relation between ξ and χ, i.e., on the shape of the specimen and the magnitude of K_1. For $\eta > \eta_{\rm cr1}$ the dependence $a^2(\omega, \eta)$ becomes multivalued, and the possibility of an amplitude jump appears—this is the region of the first instability (the foldover-effect region *). For still larger η the resonance curve begins to bend in the opposite direction, and for $\eta > \eta_{\rm cr2}$ a double jump of the amplitude is possible (double foldover effect). From Fig. 1 it is seen that instability may appear both in the region of small and in that of large amplitudes of the stationary oscillations, in other words, at small and large precession angles.

* The term foldover was proposed by Gottlieb [(4)].

The critical values of the amplitudes at which the indicated instabilities arise in the motion of the magnetization can be found from the condition $d\omega/da^2 = 0$, which leads to the equation

$$[\omega - \nu(a)]^2 - 2a^2(1-a^2)\frac{d\nu(a)}{da^2}[\omega - \nu(a)] + a^2\omega^2(1-a^2)^2 = 0. \tag{6}$$

However, in explicit form equation (6) is cumbersome and inconvenient for analysis. To estimate the value $a_{\rm cr}$ in the case of small amplitudes, we expand the $\sqrt{1-a^2}$ contained in (6), and restrict ourselves to powers of a no higher than the fourth. From the requirement that the roots of the resulting equation be real, we find

$$a^2_{\rm cr\,1} = \frac{2}{\beta^2}\left(\sqrt{\beta^2+1} - 1\right)\frac{\nu + \xi - 2\chi}{|\nu + 3\xi - 20\chi|},$$

where $\beta = (9\chi - \xi)/\alpha(\nu + 3\xi - 20\chi)$. If $|\beta| \gg 1$, which is usually the case, then

$$a^2_{\rm cr\,1} = 2\alpha(\nu + \xi - 2\chi)/|9\chi - \xi|. \tag{7}$$

In the case of large amplitudes ($a^2 \lesssim 1$), as follows from (5), $\omega \simeq \nu(a)$. Then, to estimate $a_{\rm cr\,2}$, one may use the condition $d\nu(a)/da^2 = 0$, which gives

$$a^2_{\rm cr\,2} = 2(9\chi - \xi)/21\chi. \tag{8}$$

Since always $\eta^2 \geq \alpha^2\nu^2(a)a^2$ (see equation (5)), from (7) and (8) we obtain the following threshold levels of microwave power at which nonlinear effects may be observed:

$$\eta^2_{\rm cr\,1} = 2\alpha^3(\nu + \xi - 2\chi)^3(1+\alpha)^2/|9\chi - \xi|, \tag{9}$$

$$\eta_{\mathrm{cr}\,2}^2 = 2\alpha^2 \left[\nu + 7\chi \left(\frac{3\chi + 2\xi}{21\chi}\right)^{3/2}\right]^2 \frac{9\chi - \xi}{21\chi}. \tag{10}$$

Since $\alpha \simeq \gamma\Delta H/\nu$, where ΔH is the half-width of the resonance line, formula (9) agrees with the expressions obtained in ([1,2,4,6,7]).

The double instability in the motion of the magnetization vector may be the cause of a number of interesting relaxation processes. However, it is possible if $9\chi - \xi$ and χ (see (10)) have the same signs. In addition, ξ/ν, χ/ν, and α must be small quantities, since otherwise the region of the double foldover effect will become unattainable because of the great steepness of the resonance curve. The indicated requirements are simultaneously fulfilled, for example, for spherical samples made of a material with small K_1/M_0 and ΔH. The double instability will be easiest to detect if samples with $\xi = 2\chi$ are used. The resonance curves in Fig. 1 are shown precisely for this case.

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Note: Figure translations are in progress. See original paper for figures.