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Abstract

Full Text

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*Cybernetics
and Control Theory*

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ON THE FREQUENCY AND TIME CHARACTERISTICS OF SYSTEMS WITH VARIABLE PARAMETERS

(Presented by Academician B. N. Petrov on 16 X 1968)

As is known, the direct application of the Laplace transform to the study of automatic-control systems with variable parameters does not lead to simplification (except in certain special cases), since the equations in the image domain may be more complicated than in the original domain ⁽¹⁾. Therefore, as a rule, approximate methods are used, based on the determination of the adjoint transition function ^(2,3). However, rigorously these methods are applicable only to systems with slowly varying parameters and, moreover, while making it possible to compute the image of the adjoint transition function, they can be used only within the corresponding limited bounds ⁽²⁾.

Methods constructed on the basis of Hill' s work ⁽⁴⁾ have broader possibilities as applied to the study of systems with periodically varying parameters. In this case, closed-form expressions may be obtained for the determinant of the system and for the transfer function (see, for example, ⁽⁵⁾). However, for this class of systems difficulties arise in computing infinite numerical determinants; overcoming these difficulties generally requires carrying out additional investigations.

In the present paper it is shown that, in systems with monotonically varying parameters, in the case when the dependence of the parameters on time can be approximated by means of an infinite exponential series, the indicated difficulties in computing infinite numerical determinants can be avoided, since here, in contrast to systems with periodically varying parameters, a one-sided system appears instead of a two-sided infinite system of equations.

We shall consider systems whose differential equation has the form

$$\sum_{i=0}^n a_i \frac{d^i y}{dt^i} + \sum_{l=0}^{n-2} \frac{d^l}{dt^l} [f_l(t)y] = F(t), \quad (1)$$

where

$$\left. \frac{d^r y}{dt^r} \right|_{t=0} = 0 \quad \text{for } r = 0, 1, 2, \dots, n-1; \quad (2)$$

$$f_l(t) = \sum_{m=1}^{\infty} b_{lm} e^{-m\beta t}, \quad l = 0, 1, 2, \dots, n-2; \quad (2)$$

$$\sum_{m=1}^{\infty} |b_{lm}| = A_l < \infty.$$

Applying the Laplace transform to equation (1) and taking into account the shift theorem, we obtain

$$y(p) + \sum_{m=1}^{\infty} R_m(p) y(p + m\beta) = \Phi(p), \quad (3)$$

where

$$R_m(p) = \frac{Q_m(p)}{N(p)}, \quad \Phi(p) = \frac{F(p)}{N(p)}, \quad Q_m(p) = \sum_{l=0}^{n-2} b_{lm} p^l,$$

$$N(p) = \sum_{i=0}^n a_i p^i = a_n \prod_{i=1}^n (p - \alpha_i).$$

Equation (3) contains an infinite number of unknowns. To obtain a complete system, we replace p by $p + k\beta$, where $k = 0, 1, \dots, \infty$; then we obtain a system of equations of the form *

$$y(p + k\beta) + \sum_{m=1}^{\infty} R_m(p + k\beta) y[p + (k + m)\beta] = \Phi(p + k\beta), \quad (4)$$

$$k = 0, 1, 2, \dots, \infty.$$

From the system of equations (4) we have

$$y(p) = \Delta_1(p) / \Delta(p). \quad (5)$$

In our case

$$\Delta(p) \equiv 1, \quad (6)$$

since the matrix $\Delta(p)$ is triangular and all its diagonal elements are equal to one. Taking identity (6) into account, we obtain

$$y(p) = \Delta_1(p) = \begin{vmatrix} \Phi(p) & R_1(p) & R_2(p) & R_3(p) & \dots \\ \Phi(p + \beta) & 1 & R_1(p + \beta) & R_2(p + \beta) & \dots \\ \Phi(p + 2\beta) & 0 & 1 & R_1(p + 2\beta) & \dots \\ \Phi(p + 3\beta) & 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots \end{vmatrix}. \quad (7)$$

The determinant (7) is a meromorphic function having poles at the points $p = \alpha_i - k\beta$ ($i = 1, \dots, n; k = 0, \dots, \infty$), as well as the poles generated by the function $F(p) = \mathcal{L}[F(t)]$, which we denote by $p = \gamma_s - k\beta$ ($s = -\infty, \dots, 0, \dots, \infty; k = 0, \dots, \infty$).

As can be seen,

$$\lim_{p \rightarrow \infty} \Delta_1(p) = 0,$$

and, if all poles are assumed simple, then

$$\Delta_1(p) = y(p) = \sum_{i=1}^n \sum_{k=0}^{\infty} \frac{C_{ik}}{p - \alpha_i + k\beta} + \sum_{s=-\infty}^{\infty} \sum_{k=0}^{\infty} \frac{C_{sk}}{p - \gamma_s + k\beta}, \quad (8)$$

where

$$C_{ik} = \lim_{p \rightarrow \alpha_i - k\beta} [\Delta_1(p)(p - \alpha_i + k\beta)].$$

If from the image (8) we pass to the original $y(t)$, then one may write

$$y(t) = \sum_{i=1}^n \sum_{k=0}^{\infty} C_{ik} e^{(\alpha_i - k\beta)t} + \sum_{s=-\infty}^{\infty} \sum_{k=0}^{\infty} C_{sk} e^{(\gamma_s - k\beta)t}. \quad (9)$$

It is interesting to note that replacing p in (3) by $p + k\beta$, where $k = -\infty, \dots, \infty$, does not change the solutions (8) and (9), since in this case as well $\Delta(p) \equiv 1$, and $\Delta_1(p)$ is easily reduced to the form of determinant (7).

For systems with variable parameters, the determination of the impulse response function is important, i.e., the response of a previously unexcited

* It should be noted that the system of recurrence equations to which the equation of a system with variable parameters is reduced can also be regarded as the

equation of a certain equivalent multidimensional system with unknown parameters and, accordingly, theorems on invariant multidimensional systems can be applied.

system to an input signal in the form of a delta function. To determine the impulse response function, set in (1)

$$F(t) = \delta(t - \xi),$$

then

$$\Phi(p) = \frac{e^{-\xi p}}{N(p)},$$

$$y(p) = g(p, \xi) = e^{-\xi p} \begin{vmatrix} \frac{1}{N(p)} & \frac{Q_1(p)}{N(p)} & \frac{Q_2(p)}{N(p)} & \dots \\ \frac{e^{-\xi\beta}}{N(p+\beta)} & 1 & \frac{Q_1(p+\beta)}{N(p+\beta)} & \dots \\ \frac{e^{-2\xi\beta}}{N(p+2\beta)} & 0 & 1 & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix}.$$

Assuming that $-(\alpha_i - \alpha_j)/\beta \neq k$ ($i \neq j$; $i, j = 1, 2, \dots, n$; $k = 0, 1, \dots, \infty$), we obtain

$$g(p, \xi) = e^{-\xi p} \sum_{i=1}^n \sum_{k=0}^{\infty} C_{ik}(\xi) (p - \alpha_i + k\beta)^{-1},$$

or, passing to the original,

$$g(t, \xi) = \sum_{i=1}^n \sum_{k=0}^{\infty} C_{ik}(\xi) e^{(\alpha_i - k\beta)(t - \xi)}, \quad (10)$$

where

$$C_{ik}(\xi) = A_{ik} e^{-k\beta\xi} c_{i0}(\xi). \quad (11)$$

A_{ik} in expression (11) is a determinant of order k , composed as follows:

$$A_{ik} = (-1)^k \begin{vmatrix} \frac{Q_1(\alpha_i - k\beta)}{N(\alpha_i - k\beta)} & \frac{Q_2(\alpha_i - k\beta)}{N(\alpha_i - k\beta)} & \cdots & \frac{Q_k(\alpha_i - k\beta)}{N(\alpha_i - k\beta)} \\ 1 & \frac{Q_1[\alpha_i - (k-1)\beta]}{N[\alpha_i - (k-1)\beta]} & \cdots & \frac{Q_{k-1}[\alpha_i - (k-1)\beta]}{N[\alpha_i - (k-1)\beta]} \\ 0 & 1 & \cdots & \frac{Q_{k-2}[\alpha_i - (k-2)\beta]}{N[\alpha_i - (k-2)\beta]} \\ 0 & 0 & \cdots & \frac{Q_1(\alpha_i - \beta)}{N(\alpha_i - \beta)} \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix}.$$

Taking equality (11) into account, we obtain

$$g(t, \xi) = \sum_{i=1}^n C_{i0}(\xi) e^{\alpha_i(t-\xi)} \sum_{k=0}^{\infty} A_{ik} e^{-k\beta t}, \quad (12)$$

$$g(t, \xi) \equiv 0 \quad \text{for } t < \xi \text{ and } \xi < 0.$$

From the foregoing it is clear that the analysis of systems described by equations of the form (1) can be carried out, with the required accuracy, by the methods used in the analysis of systems with constant parameters.

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Note: Figure translations are in progress. See original paper for figures.

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