ALBEDO OF A PLANETARY ATMOSPHERE

Astronomy

1969

SovietRxiv

Abstract

Full Text

UDC 523.035.2

Astronomy

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The purpose of the present note is to obtain formulas for the plane and spherical albedo of a planet surrounded by an atmosphere of infinitely large optical thickness. The scattering indicatrix $x(\gamma)$ is assumed arbitrary, and the quantity λ, representing the ratio of the scattering coefficient to the sum of the scattering and true-absorption coefficients, is close to unity (i.e., $1 - \lambda \ll 1$).

Let the cosine of the angle of incidence of the solar rays at the given point be equal to ζ, and let the illumination of the upper boundary of the atmosphere produced by them be $\pi S \zeta$. We represent the azimuth-averaged intensity of radiation diffusely reflected by the atmosphere at an angle $\arccos \eta$ to the normal in the form

$$I(\eta, \zeta) = S\rho(\eta, \zeta)\zeta. \tag{1}$$

Then the plane albedo of the atmosphere is defined by the formula

$$A(\zeta) = 2\int_0^1 \rho(\eta, \zeta)\eta\, d\eta, \tag{2}$$

and the spherical albedo of the planet is equal to

$$A_* = 2\int_0^1 A(\zeta)\zeta\, d\zeta. \tag{3}$$

The quantities $A(\zeta)$ and A_* can be expanded in a series in powers of $\sqrt{1-\lambda}$. Earlier ([1]), the zeroth and first terms of these expansions were obtained. Now, by the same method, we shall also find the term of order $1 - \lambda$.

We shall proceed from the following two relations connecting with one another the reflection coefficient of the atmosphere $\rho(\eta, \zeta)$, its transmission coefficient $u(\eta)$, and the relative radiation intensity $i(\eta)$ in the deep layers of the medium:

$$i(-\zeta) = 2\int_0^1 \rho(\eta, \zeta) i(\eta)\eta\, d\eta, \tag{4}$$

$$Mu(\zeta) = i(\zeta) - 2\int_0^1 \rho(\eta,\zeta) i(-\eta)\eta \, d\eta. \tag{5}$$

Here

$$M = 2\int_{-1}^1 i^2(\eta)\, d\eta, \tag{6}$$

and the functions $u(\eta)$ and $i(\eta)$ are normalized according to the conditions

$$2\int_0^1 u(\eta) i(\eta)\eta \, d\eta = 1, \tag{7}$$

$$\frac{\lambda}{2}\int_{-1}^1 i(\eta)\, d\eta = 1. \tag{8}$$

To determine the function $i(\eta)$, one uses the integral equation obtained by V. A. Ambartsumian ([2]). From this equation we find

$$i(\eta) = 1 + 3\eta\sqrt{\frac{1-\lambda}{3-x_1}} + \frac{15\eta^2 - x_2}{5 - x_2}(1-\lambda) + \dots, \tag{9}$$

where x_1 and x_2 are the first and second coefficients of the expansion of the scattering indicatrix $x(\gamma)$ in Legendre polynomials.

The function $u(\eta)$, in accordance with one of the results of work ([1]), can be represented in the form

$$u(\eta) = u_0(\eta)\left(1 - C\sqrt{1-\lambda}\right) + \dots, \tag{10}$$

where $u_0(\eta)$ is the atmospheric transmission coefficient in the case of pure scattering and C is a constant. Substituting (9) and (10) into (7), we obtain

$$C = \frac{6}{\sqrt{3-x_1}}\int_0^1 u_0(\eta)\eta^2\, d\eta. \tag{11}$$

Put

$$A(\xi) = 1 + A_1(\xi)\sqrt{1-\lambda} + A_2(\xi)(1-\lambda) + \dots, \tag{12}$$

$$\rho(\eta,\xi) = \rho_0(\eta,\xi) + \rho_1(\eta,\xi)\sqrt{1-\lambda} + \rho_2(\eta,\xi)(1-\lambda) + \dots, \tag{13}$$

where $\rho_0(\eta, \xi)$ is the reflection coefficient of the atmosphere for $\lambda = 1$. The quantity $\rho_0(\eta, \xi)$ satisfies the condition

$$2\int_0^1 \rho_0(\eta, \xi)\eta\, d\eta = 1. \tag{14}$$

In addition, we have

$$2\int_0^1 \rho_1(\eta, \xi)\eta\, d\eta = A_1(\xi), \qquad 2\int_0^1 \rho_2(\eta, \xi)\eta\, d\eta = A_2(\xi). \tag{15}$$

Substitution of expressions (9), (10), (12), and (13) into relations (4) and (5) gives

$$A_1(\xi) = -\frac{4}{\sqrt{3 - x_1}}\, u_0(\xi), \tag{16}$$

$$A_2(\xi) = \frac{15}{5 - x_2} v_0(\xi) + \frac{4C}{\sqrt{3 - x_1}}\, u_0(\xi), \tag{17}$$

where

$$u_0(\xi) = \frac{3}{4}\left[\xi + 2\int_0^1 \rho_0(\eta, \xi)\eta^2\, d\eta\right], \tag{18}$$

$$v_0(\xi) = \xi^2 - 2\int_0^1 \rho_0(\eta, \xi)\eta^3\, d\eta. \tag{19}$$

Introducing (16) and (17) into (12) and denoting

$$D = 24\int_0^1 u_0(\eta)\eta^2\, d\eta, \tag{20}$$

we find

$$A(\xi) = 1 - 4\sqrt{\frac{1 - \lambda}{3 - x_1}}\, u_0(\xi) + \left[\frac{15}{5 - x_2} v_0(\xi) + \frac{D}{3 - x_1} u_0(\xi)\right](1 - \lambda). \tag{21}$$

This asymptotic formula determines the albedo of the atmosphere for the angle of incidence of the solar rays $\arccos \xi$.

The functions $u_0(\zeta)$ and $v_0(\zeta)$ entering into (21) satisfy the conditions

$$2\int_0^1 u_0(\zeta)\,\zeta\,d\zeta = 1, \qquad \int_0^1 v_0(\zeta)\,\zeta\,d\zeta = 0. \tag{22}$$

Substituting (21) into (3) and using (22), we obtain the following asymptotic formula for the spherical albedo of the planet:

$$A_* = 1 - 4\sqrt{\frac{1-\lambda}{3-x_1}} + D\,\frac{1-\lambda}{3-x_1}. \tag{23}$$

We see that the spherical albedo, when only the first two terms of the expansion in powers of $\sqrt{1-\lambda}$ are taken into account, depends not on the entire scattering indicatrix, but only on the parameter x_1. This property is approximately preserved also when the term of order $1-\lambda$ is taken into account, since the quantity D depends only weakly on the scattering indicatrix (one may take the value of this quantity for isotropic scattering, $D = 8.5$).

Formulas (21) and (23) are the more accurate, the smaller $1-\lambda$ is. The terms of order $1-\lambda$ obtained here often substantially increase the accuracy of determining the quantities $A(\zeta)$ and A_*. As an example, Table 1 gives values of the quantity $A(\zeta)$ for the scattering indicatrix $x(\gamma) = 1 + \cos\gamma + P_2(\gamma)$ at $\lambda = 0.99$. The approximate values were obtained from formula (21), first without the term of order $1-\lambda$, and then with this term. The exact values were calculated from formulas taken from work ([3]).

Table 1

Values of the albedo $A(\zeta)$

ζ	Approximate	Approximate	Exact	ζ	Approximate	Approximate	Exact
0	0.886	0.889	0.889	0.6	0.731	0.767	0.764
0.1	0.854	0.860	0.860	0.7	0.709	0.752	0.748
0.2	0.827	0.838	0.838	0.8	0.686	0.738	0.732
0.3	0.803	0.819	0.818	0.9	0.663	0.725	0.717
0.4	0.779	0.800	0.799	1.0	0.641	0.712	0.702
0.5	0.755	0.783	0.781				

Let us apply formula (23) to the atmosphere of Venus, considering its optical thickness to be infinitely large. It is known from observations (see ([4])) that the spherical albedo of Venus in the visual region of the spectrum is equal to 0.76. For the parameter x_1, from an analysis of polarimetric observations, the value 2.1 was recently obtained ([5]). Substituting these values of A_* and x_1 into formula (23), we find $1-\lambda = 0.0045$.

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Received
15 X 1968

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Note: Figure translations are in progress. See original paper for figures.