# A Nonparametric Method for Determining the Depth of the Top of the Crystalline Basement from Frequency-Sounding Curves

GEOPHYSICS

1968

SovietRxiv

Fig. 1

Figure 1: Fig. 1

## Abstract

## Full Text

UDC 550.837

**GEOPHYSICS**

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# A Nonparametric Method for Determining the Depth of the Top of the Crystalline Basement from Frequency-Sounding Curves

*(Presented by Academician A. V. Peive on 10 VIII 1967)*

One of the principal geological problems usually posed for the method of frequency soundings (FS) is the determination of the depth and relief of the top of the crystalline basement. The FS curves obtained under these conditions reflect various multilayer geoelectric structures with a marker horizon $\mu_n \to \infty$, and their right-hand branches can correspond to only two types of three-layer curves: $H$ or $A$.

We shall explain the essence of the proposed method using as an example the amplitude curves for the vertical component of the magnetic field ($B_z$). Let us first turn to two-layer curves of type $\mu_2 \to \infty$. Analysis of these curves, both for the magnetic and for the electric components of the field, makes it possible to establish a number of relationships between the parameters of the two-layer section, the spacing with which the sounding was obtained, and the characteristic points of the curve.

Curve $I$ in Fig. 1 depicts the relationship between the amplitudes $A$ of the right-hand branches of the FS curves for the component $B_z$ and the quantities $r/H_1$; $r$ is the spacing. $H_1$ is the thickness of the upper layer. Curve $II$ characterizes the dependence of the abscissae $\lambda_1/H_1$ of the maxima of the FS curves for the component $B_z$ on $r/H_1$; $\lambda_1$ is the wavelength in the upper layer. Curve $III$ characterizes the dependence of the abscissae $\lambda_1/H_1$ of the minima of the curves on $r/H_1$.

**Fig. 1**

It follows from Fig. 1 that, in the case of a two-layer section of type $\mu_2 \to \infty$, determining the thickness of the upper layer is extremely simple: the amplitude of the right-hand branch of the FS curve obtained (from the axis $\rho = \rho_1$ to its maximum) is measured in centimeters, and from curve $I$ the value of $r/H_1$ is found. Since $r$ is known, $H_1$ becomes known.

The dependence between the quantities $A$, $(\lambda_1/H_1)_{\max}$, and $(\lambda_1/H_1)_{\min}$ on $r/H_1$ can be represented analytically with an error of up to 1%.

Between $A$ and $r/H_1$ there is the dependence

$$A = 1.71\,(r/H_1 - 1.7)^{0.63}, \tag{1}$$

whence

$$H_1 = 2.33r/(A^{1.59} + 3.96). \tag{2}$$

Taking into account that FS curves are plotted on a double logarithmic scale and taking the construction modulus equal to 1 : 10, we find

$$A = 10(\lg\rho_m - \lg\rho_1) = 10\lg(\rho_m/\rho_1), \tag{3}$$

where $\rho_m$ is the value of $\rho$ at the maximum of the right branch of the curve, and $\rho_1$ is the longitudinal resistivity of the upper layer.

From (2) and (3) we obtain

$$H_1 = 2.33r/[(10\lg\rho_m/\rho_1)^{1.59} + 3.96]. \tag{4}$$

Between $(\lambda_1/H_1)_{\max}$ and $r/H_1$ there is the dependence

$$(\lambda_1/H_1)_{\max} = 2.95(r/H_1)^{0.6}. \tag{5}$$

Taking into account that

$$(\lambda_1/H_1)_{\max} = \frac{3.16\sqrt{\rho_1}}{H}\left(\frac{1}{\sqrt{f}}\right)_{\max},$$

we find

$$H_1 = 1.18\rho_1^{1.25}(1/\sqrt{f})^{2.5}_{\max}/r^{1.5}. \tag{6}$$

Between $(\lambda_1/H_1)_{\min}$ and $r/H_1$ there is the dependence

$$(\lambda_1/H_1)_{\min} = 2.1(r/H_1)^{0.4}. \tag{7}$$

Since

$$(\lambda_1/H_1)_{\min} = \frac{316\sqrt{\rho_1}}{H_1}\left(\frac{1}{\sqrt{f}}\right)_{\min},$$

then

$$H_1 = 1.97\rho_1^{0.83}(1/\sqrt{f})_{\min}^{1.67}/r^{0.67}. \quad (8)$$

The quantity $H_1$ is related to the longitudinal conductance ($S_1$) and $\rho_1$ by the known relation

$$H_1 = \rho_1 S_1. \quad (9)$$

For two-layer and multilayer theoretical and experimental FS curves, the quantities $S_1$, $(1/\sqrt{f})_{\max}$, and $(1/\sqrt{f})_{\min}$ are determined from the curve itself, and the value of $r$ is known; therefore, the correct determination of $H_1$ depends on the correct choice of $\rho_1$.

Each of formulas (4), (6), (8), and (9) characterizes different laws of variation of $H_1$ as a function of the variation of $\rho_1$. They give identical values of $H_1$ only when $\rho_1$ is chosen to correspond to the true value; otherwise, the resulting $H_1$ are different.

Any multilayer FS curve reflecting a section with a reference horizon having $\mu_n \to \infty$ can be represented by an equivalent two-layer curve in which $\rho_1$ is equal to the average longitudinal resistivity of all layers of the section ($\rho_l$), and $H_1$ is equal to the total thickness ($H$) of all layers of the section down to the reference horizon. Therefore, everything said concerning two-layer curves applies equally to multilayer curves in which the lower layer has $\mu_n \to \infty$.

This makes it possible to interpret multilayer FS curves of the type $\mu_n \to \infty$ on the basis of the obtained formulas (4), (6), (8), and (9) by testing different values of $\rho_l$. In this case, the true value of $H$ will be the one obtained identically by all formulas. At the same time, the true value of $\rho_l$ is also determined.

It should be noted that each of formulas (4), (6), (8), and (9) has a different resolving power, expressed in the fact that a change in $\rho_l$ by a given amount leads to different changes in the value of $H$. On average, when $\rho_l$ is changed by 100%, the discrepancies between the values of $H$ obtained by the different formulas reach 25–30%.

It appears possible, however, to increase the resolving power of the method and to bring it on average to 100% instead of 25–30%. This is achieved by various combinations among the obtained formulas.

It proves advantageous to use formulas (10) and (11), of which the first was obtained by combining (4), (8), and (9), while the second is the result of combining (8) and (9):

Fig. 2

Figure 2: Fig. 2

$$H = 4{,}59\, r^{0{,}33}\, (1/\sqrt{f})^{1{,}6}_{\min} / \left[(10 \lg \rho_m/\rho_l)^{1{,}59} + 3{,}96\right]\, \rho_l^{0{,}17} S; \qquad (10)$$

$$H = 0{,}51\, S^2 \rho_l^{1{,}17} r^{0{,}67} / (1/\sqrt{f})^{1{,}67}_{\min}. \qquad (11)$$

It follows from formulas (10) and (11) that, in order to carry out the interpretation, it is necessary to use the known value of $r$ and to determine from the obtained frequency-sounding curve the quantities $S$, $\rho_m$, and $(1/\sqrt{f})_{\min}$; moreover, if the first three of these are constant for the curve, the fourth will vary depending on the value of $\rho_l$ chosen for interpretation.

The value $(1/\sqrt{f})_{\min}$ is determined as follows. The horizontal axis $\rho = 1$ of the two-layer palette $\mu_2 = \infty$ is superposed with the value of $\rho_l$ chosen for interpretation, and one notes with which palette curve the maximum of the curve being interpreted completely coincided. The sought value $(1/\sqrt{f})_{\min}$ is the abscissa of the FS curve with which the abscissa of the minimum of the palette curve coincided.

**Fig. 2**

Let us turn to several illustrations. In Fig. 2, $I$ represents a three-layer FS curve for the component $B_z$ with values $\mu_2 = 1/4$ and $\nu_2 = 1$, $\mu_3 = \infty$ ($r = 10$ km). It can be represented by an equivalent two-layer curve for which $\rho_1 = 0{,}4\ \Omega \cdot \mathrm{m}$ and $H_1 = 2$ km.

Suppose, as is the case in reality, that the parameters of the section characterized by this curve are unknown to us, and only the sounding length $r$ is known. We determine from the curve the value $S$ by the known method (1) ($S = 5$) and carry out its interpretation by successive trials of various values of $\rho_l$, determining from formulas (10) and (11) the corresponding values of $H$. The data obtained are collected in Table 1, from which it is seen that the values of $H$ are obtained correctly and coincide with one another by both formulas (1.92 km instead of 2 km) when $\rho_l$ is close to the true value ($0{,}4\ \Omega \cdot \mathrm{m}$). For other values of $\rho_l$, the values of $H$ do not coincide with one another, and none of them corresponds to reality.

**Table 1**

| | 0,26 | 0,29 | 0,33 | 0,47 | 0,58 | 0,58 |
|---|---|---|---|---|---|---|
| $\rho_l$ | 0,26 | 0,29 | 0,33 | 0,47 | 0,58 | 0,58 |
| $H$ (10) | 1,17 | 1,33 | 1,62 | 2,07 | 2,73 | 3,63 |
| $H$ (11) | 1,38 | 1,54 | 1,71 | 1,95 | 2,3 | 2,82 |

| | 0,26 | 0,29 | 0,33 | 0,47 | 0,58 | 0,58 |
|---|---|---|---|---|---|---|
| Discrepancy between (10) and (11), in % | 18 | 16 | 6 | 3 | 19 | 29 |

In Fig. 2 the curve $II$ characterizes a four-layer section of type $KH$ ($\mu_2 = \infty$, $\nu_2 = 1/4$, $\mu_3 = 1/4$, $\nu_3 = 2$, $\mu_4 = \infty$), $r = 16$ km. Table 2 contains the results of the interpretation. The obtained values for $H = 3{,}25$ and 3,3 km ($\rho_l = 0{,}35\ \Omega \cdot \mathrm{m}$) agree well with their actual values, equal to 3.25 km.

Table 2

| $\rho_l$ | 0.2 | 0.25 | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 |
|---|---|---|---|---|---|---|---|---|---|
| $H$ (10) | 1.25 | 1.9 | 2.58 | 3.25 | 3.85 | 4.65 | 5.48 | 6.4 | 7.2 |
| $H$ (11) | 2.48 | 2.7 | 2.95 | 3.3 | 3.65 | 4.1 | 4.45 | 4.8 | 5.15 |
| Discrepancy between (10) and (11), % | 100 | 42 | 14 | 1.5 | 5.5 | 13 | 22 | 35 | 40 |

It is important that the proposed procedure makes it possible to carry out a parameter-free interpretation of multilayer frequency-sounding curves ending in a three-layer part of type $A$, which is not accessible to existing methods. This assertion requires no special proof, since on the right branch of a three-layer curve of type $A$ one can find all the required quantities ($\rho_m$, $(1/\sqrt{f})_{\max}$, $(1/\sqrt{f})_{\min}$, and $S$) needed to determine the quantities $H$ and $\rho_l$. At the same time, it is naturally necessary that the resistivity of the overlying bed not be equal or close to the resistivity of the reference horizon.

Nevertheless, an extensive check was carried out, fully confirming the possibility of parameter-free interpretation of curves of this type.

As an illustration, Fig. 2 III shows a three-layer curve of type $A$ ($\rho_1 = 1$ ohm • m, $H_1 = 1$ km, $\mu_2 = 2$, $\nu_2 = 2$, $\mu_3 = \infty$, $\nu_3 = \infty$, $r = 16$ km).

Table 3

| $\rho_l$ | 0.8 | 1 | 1.2 | 1.5 | 1.6 | 1.8 | 2.3 |
|---|---|---|---|---|---|---|---|
| $H$ (10) | 1.3 | 1.78 | 2.34 | 2.93 | 3.44 | 4.5 | 6.1 |

| $\rho_l$ | 0.8 | 1 | 1.2 | 1.5 | 1.6 | 1.8 | 2.3 |
|---|---|---|---|---|---|---|---|
| $H$ (11) | 2.00 | 2.35 | 2.7 | 3.05 | 3.11 | 3.45 | 4.21 |
| Discrepancy between (10) and (11), % | 54 | 32 | 15 | 4 | 11 | 31 | 47 |

Table 3 gives the results of the interpretation. The values $H = 2.93$ and 3.05 km ($\rho_l = 1.5$ ohm · m), obtained close to one another, correspond to their true values.

From Tables 1, 2, and 3 one can see the resolving power of the method when formulas (10) and (11) are used.

To determine the limits of applicability of the described method, it is necessary to carry out additional calculations of theoretical FS curves for comparison with those already available.

However, in general it appears clear that the method will fail to give satisfactory accuracy only for those groups of geoelectric structures for which, for a given spacing (taking account of measurement error), the coordinates of the right-hand branches of the FS curves (including the maximum) turn out to be identical.

In conclusion, we note that, besides the combinations (10) and (11) given in the article, other combinations of the initial formulas ((4), (6), (8), and (9)) are also possible; depending on the geoelectric parameters of the section, in the interpretation of the sounding curves obtained they may yield results no less satisfactory.

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Received
8 VIII 1967

## CITED LITERATURE

1. B. S. Epshtein, *Izv. AN SSSR*, geophys. ser., No. 12 (1957).

*Note: Figure translations are in progress. See original paper for figures.*