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# EMISSION REGIMES OF A GAS LASER WITH A RING RESONATOR

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**Abstract**

**Full Text**

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**PHYSICS**

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**EMISSION REGIMES OF A GAS LASER WITH A RING RESONATOR**

*(Presented by Academician N. G. Basov, 23 VI 1967)*

1. Recently a number of works have appeared on the interaction of traveling waves in a rotating ring laser (<sup>1-4</sup>). Despite the progress achieved, a number of fundamental questions remain unclear, questions of not only technical but also purely physical interest. Among such questions, in our view, the greatest interest is presented by the effect of spatial modulation of the active medium by the field itself, which has a substantial effect on the laser emission regimes; by the influence of the interaction of oscillation types with different longitudinal numbers; and, as experiment shows, by the dependence of the locking band on the resonator quality factor (<sup>1</sup>). In this connection, contrary to the increase of the locking band with increasing resonator damping expected from radio-engineering analogies, the locking band, on the contrary, decreases sharply.

In the present work all the above-mentioned effects are considered from a unified point of view.

2. The time components of the laser field

$$E(x, t) = E_1(t) \exp i[\nu t + \varphi_1(t) - kx] + E_2(t) \exp i[\nu t + \varphi_2(t) + kx] \quad (1)$$

in the rotating coordinate system satisfy the equations

$$\ddot{\varepsilon}_i + \frac{\nu}{Q_i} \dot{\varepsilon}_i + \frac{\nu}{G_i} \dot{\varepsilon}_j + \omega_i^2 \varepsilon_i = -4\pi \ddot{p}_i; \quad i, j = 1, 2; \quad i \neq j. \quad (2)$$

Here  $E_i, \varphi_i, Q_i$  are the amplitude, phase, and quality factor for the  $i$ -th wave;  $k$  is the wave number;  $\omega_i$  are the wave frequencies depending on rotation;  $x \in [0, L]$  is the coordinate along the resonator axis;  $G_i$  are coefficients taking into account the exchange of energy of the traveling waves due to scattering at elements of the resonator.

For the amplitudes and phases of the field (1), slowly varying in comparison with the frequencies  $\omega_i$ , system (2) takes the form

$$\dot{E}_i + \frac{1}{2} \frac{\nu}{Q_i} E_i + \frac{1}{2} \frac{\nu}{G_i} E_j \cos \varphi = \frac{1}{2} \frac{\nu}{\varepsilon} \sqrt{\pi} A e^{-\xi^2} E_i \{1 - I_i - I_j + I_j f\}; \quad (3)$$

$$\dot{\varphi} = \omega + \frac{\nu}{2} \sin \varphi \left\{ \frac{1}{G_1} \frac{E_2}{E_1} + \frac{1}{G_2} \frac{E_1}{E_2} \right\} + \frac{1}{2} \frac{\nu}{\varepsilon} A \frac{\xi}{\eta} L(\xi) Z_i(\xi) [I_1 - I_2]. \quad (4)$$

Here  $\varphi = \varphi_2 - \varphi_1$ ;  $\omega = \omega_2 - \omega_1$ ;  $f = -\eta^2 + \xi^2/(1 + \xi^2)$ ;  $\eta = \gamma_{ab}/ku$ ;  $\xi = (\nu - \omega_\Lambda)/\gamma_{ab}$ , while  $\gamma_{ab}$ ,  $ku$ , and  $\omega_\Lambda$  are, respectively, the natural and Doppler line widths and the central frequency of the atomic transition. The remaining notation is the same as in (2).

For  $\omega = G_i^{-1} = 0$ , system (3)–(4) has two qualitatively different generation regimes<sup>(1)</sup>, separated by the condition  $f = 0$ . If  $|\xi| < |\xi|_{\text{cr}}$  ( $\xi_{\text{cr}}^2 = \eta^2/(1 - \eta^2)$ ), it follows from (3) that the traveling-wave regime will be stable, while for  $|\xi| > |\xi|_{\text{cr}}$  the standing-wave regime will be stable. For  $\omega, G_i^{-1} \neq 0$ , the indicated solutions are deformed, and at the same time the region of their stability also changes. Let us introduce the notation

$$r = \frac{\mu_1 - \mu_2}{\mu} (\rho - 1)^{-1}, \quad \Delta = \frac{\xi}{\eta} (1 + \xi^2)^{-1},$$

$$\Omega = \omega [\mu(\rho - 1)]^{-1},$$

where  $\rho$  is the excess of the pumping power over threshold, and

$$\mu_i = \frac{1}{2} \frac{\nu}{Q_i}, \quad \mu = \frac{1}{2} (\mu_1 + \mu_2), \quad g_i = \frac{\nu}{G_i} [\mu(\rho - 1)]^{-1},$$

and let us consider the case  $\mu = \mu_1 = \mu_2$ ,  $g = g_1 = g_2$ ,  $\Delta = 0$ , meaning that the types of oscillations have equal  $Q$ , wave scattering is isotropic, and the frequency shift due to nonlinear pulling of the modes is small compared with the shift due to the reflected energy\*.

The stability regions *I* and *II* of the stationary solutions in the indicated case are shown in Fig. 1. Region *I* corresponds to the regime of a purely standing wave, *II* to that of a traveling wave\*\* . Increasing the coupling coefficients  $g$  leads to an expansion of region *I* at the expense of a narrowing of regions *II* and *III*. For  $|g| > \eta^2$  (the condition is always satisfied as the pump approaches threshold) and  $\Omega = 0$ , region *II* shrinks to zero and the standing-wave regime becomes stable for all frequency detunings  $\xi$ .

Region *III* corresponds to the generation regime

$$\begin{aligned} \dot{\varphi} &\simeq \Omega + g \sin \varphi; & E_1 &\simeq E_2; \\ & & |\Omega| &> |g|, \end{aligned} \quad (5)$$

Fig. 1

Figure 1: Fig. 1

describing the splitting of the frequencies of the traveling waves due to rotation of the resonator. The critical detuning  $\Omega_{\text{cr}}(0) = |g|$  in a completely symmetric scheme does not depend on the effects of spatial modulation of the medium, despite the fact that the intensity of wave scattering by the periodic structure created by the field may considerably exceed the intensity of scattering at the resonator mirrors. Mathematically this fact is described by the absence of a phase coupling in (3)–(4) through terms nonlinear in the field. Physically, the reason for the effect is that the scattering centers induced by the field are not stationary with respect to the resonator of the generator. Allowance for the Doppler frequency shift of the scattered photon thus leads only to an energy coupling of the equations.

**Fig. 1**

Including  $\Delta \neq 0$  in (3)–(4) does not deform region *I*, whereas region *II*, while always remaining bounded by the straight line  $|\xi| = |\xi|_{\text{cr}}$ , expands because of the shift of  $\xi_{\Lambda}$  toward larger  $|\xi|$ . The point  $\xi_{\Lambda}$  satisfies the equation  $f = g$  for  $|\Delta| < |g|$ ,  $f = |\Delta|(2|g|/|\Delta| - 1)$  for  $|\Delta| > |g|$ , and  $f = 0$  for  $|\Delta| > 2|g|$ . Thus hysteretic regimes also become possible.

In an asymmetric scheme the effects of spatial hole burning can substantially affect the value of  $\Omega_{\text{cr}}$ . Let us consider the interaction of waves with different damping coefficients  $\mu_i$ . In this case  $\Omega_{\text{cr}}(r/f)$  (the dashed curve in Fig. 1) may appreciably exceed  $\Omega_{\text{cr}}(0)$ . A decrease in the ratio  $|r/f|$  should also lead to a decrease in  $\Omega_{\text{cr}}(r/f)$ . In particular, increasing the damping  $\mu_i$  of each of the waves under the condition  $\mu_1 - \mu_2 \simeq \text{const}$  (this is not difficult to achieve by increasing the transparency of the mirrors) at frequencies close to  $\omega_{\Lambda}$  sharply decreases  $\Omega_{\text{cr}}(r/f)$  <sup>(1)</sup>.

Similarly, at fixed  $\xi$ ,  $\Omega_{\text{cr}}$  should be affected by gas heating (with increasing  $ku$ ,  $f$  increases) and by the multimode nature of the radiation regime. In the latter case the spatial arrangement—

\* The latter is always true for a small excess over the generation threshold.

\*\* With the introduction of the coefficients  $g$ , a purely traveling-wave regime does not exist. The presence of scattering centers inevitably leads to generation in counterpropagating waves with different intensities, whose discrimination increases as  $|\xi|$  or  $|g|$  decreases.

of modes is described by the equation

$$\frac{d}{dt} [\theta_k - \theta_m] = S \sin 2[\theta_k - \theta_m] \quad (6)$$

with a coefficient  $S$  depending on the intensity of the fields, their frequency

detuning, and the spatial overlap ( $S \sim \sin(k - m)l/(k - m)l$ ) of the types of oscillations. In deriving (6) we used the equation  $\dot{\theta}_k \sim \langle P \sin(\nu_k t + \varphi_k) \sin(kx + \theta_k) \rangle$ , which follows from (2) under the condition that the coordinate dependence of the axial modes is described by the functions  $\cos(kx + \theta_k)$  and  $\cos(mx + \theta_m)$ , while the pumping, constant for  $-l/2 \leq x \leq l/2$ , is equal to zero outside this interval. For sufficiently small  $l$ ,  $S > 0$ , and the stable solution of (6) is  $|\theta_k - \theta_m| = \pi/2$ , which is also confirmed by experiment<sup>(5)</sup>. In this case, for  $x \in [-l/2, +l/2]$ , the antinodes of one mode coincide with the nodes of the other (which, of course, is impossible for  $l = L$ ); the inhomogeneities of the inversion are smoothed out ( $f - \xi^2/(1 + \xi^2)$  in (3) is reduced in comparison with  $\eta^2$ ). At the same time the region II narrows and, for fixed  $\xi$ ,  $\Omega_{cr}$  decreases.

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*Note: Figure translations are in progress. See original paper for figures.*

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