DYNAMIC PROGRAMMING AND NONLINEAR PROBLEMS OF THE STATICS OF THIN RODS

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Abstract

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DYNAMIC PROGRAMMING AND NONLINEAR PROBLEMS OF THE STATICS OF THIN RODS

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1. One of the fundamental problems of modern optimal-control theory is the minimization of the functional

$$I(y) = \int_0^T g(x, y)\, dt, \tag{1}$$

where $x \in X$ is an n-dimensional vector of phase-state functions; $y \in Y$ is an m-dimensional control vector (X, Y are prescribed sets), under the condition that x and y are related by

$$dx/dt = h(x, y), \tag{2}$$

$$x(0) = c. \tag{3}$$

Such problems can be solved by means of the classical calculus of variations, the maximum principle ([1]), and dynamic programming ([2]).

The method of dynamic programming, which is based on Bellman's "principle of optimality" as applied to multistage control processes, makes it possible directly to determine the optimal policy $v \in Y$ as the solution of the system of functional equations ([2])

$$f_N(c) = \min_{\{v\}} \left[g(c, v) + f_{N-1}(c + \Delta h(c, v)) \right], \tag{4}$$

where

$$f(c) = \min_{\{y_k = v\}} [I(y) \approx$$

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

$$\approx \sum_{k=1}^{N} g(x_k, y_k)\Delta \Bigg] , \tag{5}$$

$$N\Delta = T. \tag{6}$$

Fig. 1

2. The purpose of the present article is to show that the method of dynamic programming, readily realizable on modern electronic computers, can be applied to the solution of certain problems in the mechanics of deformable bodies.

As the object of investigation, certain nonlinear problems of the statics of thin rods ([3]) have been selected, which can also be considered

as a problem of mathematical optimization (1)–(3), if one proceeds, for example, from consideration of the minimum of the potential energy of a deformed rod under the action of forces.

Consider a cantilever curved flexible rod loaded by a load of arbitrary form (Fig. 1). Let the moment and force distributed loads, for generality, vary continuously both in magnitude and in direction along the axis of the rod, i.e.,

Fig. 2

$$m = m(s), \qquad q = q(s), \qquad \delta_2 = \varepsilon(s), \tag{7}$$

where $0 \leq s \leq 1$.

The expression for the potential energy in this case has the form

$$E = \int_0^1 \left[0.5(\varphi')^2 + K\varphi - 0.5K^2 + \overline{P}_1 \cos(\varphi - \delta_1) - \overline{m} + \overline{P}_2 \cos(\varphi - \delta_2)\right] ds, \tag{8}$$

where φ is the angle made by the tangent at any point of the bent axis of the rod with Oy (Fig. 1); $K = K(s) = 1/R$ is the variable curvature of the rod; R is the radius of curvature;

$$\overline{P}_1 = \frac{l^2}{EI} P_1, \qquad \overline{P}_2 = \frac{l^2}{EI} \int_s^1 q_\nu \, ds_\nu, \qquad \overline{m} = \frac{l^2}{EI} m,$$

and the role of the phase state is played by φ, and that of the control variable by φ'.

Denote

$$f(c) = \min_{\{\varphi'\}} E, \tag{9}$$

where $c = \varphi(0)$. Then, applying the optimality principle (2), according to (4)–(6) we obtain the functional equation of dynamic programming for the problem posed:

$$f_k(c) = \min_{\{\varphi'_k\}} \left\{ \left[0.5(\varphi'_k)^2 + K_k \varphi'_k - 0.5K_k^2 + \overline{P}_1 \cos(c - \delta_1) - \overline{m}_k + \overline{P}_2 \cos(c - \delta_2) \right] \Delta + f_{k-1}(c + \Delta\varphi'_k) \right\}, \tag{10}$$

where

$$\overline{P}_2 = \sum_{i=1}^{k} \overline{q}_i \Delta, \qquad k = 1, 2, \dots, N.$$

The solution of the resulting system is carried out on a computer, as a result of which the components of the optimal control vector $\{\varphi'_1, \varphi'_2, \dots, \varphi'_N\}$ are found. Knowing the initial state $\varphi(0)$ and using the relation

$$\varphi_{k+1} = \varphi_k + \Delta\varphi'_k, \tag{11}$$

we obtain the values of the angles of inclination of the tangents at the discrete points of the rod, i.e. $\{\varphi_0, \varphi_1, \varphi_2, \dots, \varphi_N\}$. The corresponding coordinates x_k, y_k are computed by the formulas

$$x_k = \sum_{i=1}^{k} \Delta \sin\varphi_i, \qquad y_k = \sum_{i=1}^{k} \nabla \cos\varphi_i. \tag{12}$$

3. As an illustration, the following problems were solved on the "Minsk-22" computer: a) a rectilinear rod ($\varphi(0) = 0;\ K = 0$), loaded at the end by a concentrated force $\bar{P}_1 = 5$ (Fig. 2, a); the deformed state is shown in Fig. 2, a'; b) a circular cantilever rod for $K = 1,\ \bar{P}_1 = 5,\ \bar{m} = 2.5,\ \varphi(0) = 0.1$ (Fig. 2, b, b'); c) a circular cantilever rod for $K = 1,\ \bar{q} = 5,\ \varphi(0) = 0.1$ (Fig. 2, v, v').

Fig. 3

Figure 3: Fig. 3

4. The proposed method can also be applied to the solution of the above-mentioned problems under constraints of geometric type.

Consider a thin elastic strip (Fig. 3) which is "wound" onto a curvilinear convex template, with a known equation of the contour. The formulation of the problem corresponds to work (3), i.e. it is assumed that the part of the rod already lying on the surface of the template forms a single whole with the latter, so that in the subsequent bending only its free part will participate. Using (10) and the conditions of coincidence of the points of the bent axis with the points of the template, the computer selects the magnitude $\bar{P}$ at each step, as a result of which the components $\{\bar{P}_1, \bar{P}_2, \bar{P}_3, \ldots, \bar{P}_N\}$ of the optimal force vector are obtained, necessary in order to wind the flexible rod onto the convex template. For example, for a circular template (coordinates of the center of the circle $x = 0.18984;\ y = -0.06928;\ R = 0.20208$) with $N = 10$, we obtain

$$\bar{P} = \{10, 10, 10, 15, 40, 40, 40, 40, 40, 40\}.$$

Fig. 3

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Note: Figure translations are in progress. See original paper for figures.