POSTCRITICAL ELASTIC STATES OF A CLOSED SPHERICAL SHELL

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Abstract

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The purpose of the present note is to obtain the relation between the external pressure on a closed spherical shell and its deformation after loss of stability. The deformation is assumed to be axisymmetric.

The solution of the problem posed will be based on the variational principle A (([1]), Ch. II). According to this principle, a significant postcritical deformation of an elastic shell under the action of a prescribed load is close to that form of an isometric transformation of the original surface which gives a stationary value to the functional $W = U(F) - A(F)$. This functional is defined on the isometric transformations (F) of the middle surface of the shell. The term $U(F)$ in the expression W is the deformation energy, and $A(F)$ is the work performed by the external load.

In view of the axial symmetry of the deformations under consideration of the spherical shell, the isometric transformations in question reduce to mirror buckling (([1]), Ch. III). The deformation of mirror buckling consists in the mirror reflection of a segment of the shell in the plane of its base (Fig. 1). Thus, the deformation is completely determined by specifying the angle 2α under which the buckling region is seen from the center of the shell.

For the case of not too large α, the term U of the functional W has the expression (([1]), Ch. II)

$$U = 2\pi c E (2h)^{3/2} \delta^{5/2} \frac{1}{R}, \tag{1}$$

where $2h$ is the deflection of the shell at the center of buckling, i.e., twice the height of the mirror-reflected segment; R is the radius of the shell; δ is its thickness; E is the modulus of elasticity; c is a constant ($c \simeq 0.19$). For not too large α, one may take $2h = R\alpha^2$, and the expression for U assumes the form

$$U = 2\pi c E \delta^{5/2} R^{1/2} \alpha^3. \tag{2}$$

Fig. 1

Figure 1: Fig. 1

The application of the general considerations by means of which the expression for the deformation energy (1) is obtained in the book ([1]), Ch. II, is by no means limited to the case of small values of α. On the contrary, these considerations are the more justified the larger α is, i.e., the larger the buckling of the shell. However, in the course of deriving formula (1) itself for the deformation energy, the known smallness of the parameter α was used repeatedly. Therefore, the application of formulas (1) and (2) is limited to values of α that are not too large.

The derivation of the formula for arbitrary α involves no fundamental difficulties and is based on the same considerations concerning the nature of postcritical deformations of a geometrically rigid shell (([1]), Ch. II). For the sake of brevity, we shall not present this derivation and shall formulate the final result:

For considerable bulging of a spherical shell, its deformation energy is determined by the formula

$$
\begin{gathered}
U = 2\pi(12)^{1/4} D\sqrt{R/\delta}\, I(\alpha) + \\
+ 2\pi D(1+\nu)(4\sin^2\alpha/2 - \alpha\sin\alpha).
\end{gathered}
\tag{3}
$$

Here D is the bending stiffness of the shell, ν is Poisson' s ratio,

$$I(\alpha) = \min_{\sigma} I(\sigma, \alpha),$$

$$I(\sigma,\alpha) = \sigma^3\vartheta(\alpha)/\sin\alpha + \alpha\sin\alpha/\sigma + \sqrt{2}\sigma^2(\sin\alpha - \alpha\cos\alpha)^2/\sin\alpha,$$

$$\vartheta(\alpha) = \int_0^{\alpha} (\sin t - t\cos t)^2\, dt.$$

Because of the smallness of the ratio δ/R and of the expression $4\sin^2\alpha/2 - \alpha\sin\alpha$, the second term in formula (3) may be omitted, and for the deformation energy one may take the expression

$$U = 2\pi(12^{1/4}) D\sqrt{R/\delta}\, I(\alpha). \tag{4}$$

Fig. 1

Fig. 2

Fig. 2

Figure 2: Fig. 2

For small values of α, i.e., for small bulging of the shell, the expression for the deformation energy given by this formula passes into expression (2).

The work performed by the external pressure p is

$$A = pV,$$

where V is the doubled volume of the mirror-reflected segment. Noting that

$$dA/d\alpha = p\pi R^3 \sin^3 \alpha,$$

from the stationarity condition of the functional W, i.e., from the condition

$$\frac{d}{d\alpha}(U - A) = 0,$$

we find the dependence between the external pressure p and the deformation of the shell in the state of elastic equilibrium:

$$p = 0.31\, k(\alpha) \frac{E}{1-\nu^2} \left(\frac{\delta}{R}\right)^2 \sqrt{\frac{\delta}{R}}, \qquad k(\alpha) = \frac{dI}{d\alpha} \Big/ \sin^3 \alpha.$$

Figure 2 gives the graph of the dependence of the coefficient k on the angle α. The minimum value $k \simeq 4.3$ corresponds to the angle $\alpha \simeq 60^\circ$. Substituting this value of k into the formula for p, we obtain the minimum pressure p_i capable of holding the shell in the bulged state, i.e., the lower critical pressure,

$$p_i = \frac{4}{3} \frac{E}{1-\nu^2} \left(\frac{\delta}{R}\right)^2 \sqrt{\frac{\delta}{R}}.$$

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1. A. V. Pogorelov, *Geometric Methods in the Nonlinear Theory of Elastic Shells*, "Nauka," 1967.

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