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Abstract

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Physics

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ON THE QUESTION OF THE THEORY OF RESONANT STATES

In the preceding note ⁽¹⁾, an exceptionally simple spectroscopic picture was obtained for the overwhelming majority of meson-meson ($M + M$) and meson-baryon ($M + B$) two-particle resonant states in terms of the energy variable

$$x = (M_{ik}^{\text{res}})^2 - m_i^2 - M_k^2. \quad (1)$$

It was shown that within multiplets with a given total angular momentum J and spatial parity P , the numbers x vary, as a rule, by no more than $\pm 10\%$, while the mean values $x(J^P)$ form a simple spectral sequence of the oscillator (i.e., equidistant) type.

The question arises as to the nature of the variable x and the physical meaning of its quantization. We shall now attempt to answer this question.

Our basic idea is that, in order to describe a two-particle system, it is necessary to use the well-known kinematic combination

$$\mathfrak{M} = s + u + t - 2m_i^2 - 2M_k^2, \quad (2)$$

where $s = (p_i + p_k)^2$, $u = (p'_i + p_k)^2$, $t = (p_i + p'_i)^2$. On the mass surface \mathfrak{M} vanishes.

Using the rules of quantum mechanics, expression (2) can without difficulty be given an operator meaning. The second-order differential operator obtained in this way is a successful candidate for the left-hand side of an equation describing a system of two bodies. Its important advantage in comparison, for example, with the operator of the Bethe-Salpeter equation is its explicit crossing symmetry. The property of crossing symmetry is an attribute of quantum field theory of deeply fundamental significance.

The theoretical possibility of obtaining an equation of the type

$$\mathfrak{M}\psi = 0 \quad (3)$$

was recently pointed out by V. G. Kadyshevsky ⁽²⁾, proceeding from the relativized generalization obtained by him of the quasipotential equation of Logunov-Tavkhelidze ⁽³⁾ for a system of two bodies.

It is quite clear that the problem of correctly writing an equation of type (3) is not simple and requires the solution of a number of complicated questions, such as determining the physical meaning of the function ψ , determining the operation of going off the mass surface in the case of interaction, etc. Without touching on these questions at all, we shall show that an equation of type (3), when a number of additional conditions are fulfilled, can lead to spectra in the variable $x = s - m_i^2 - M_k^2$.

Let us suppose that the solutions of equation (3) form a system of functions ψ_r describing quasibound states of the system $m_i + M_k$, which will determine the resonances of two-particle scattering. On each of these functions the differential operators s, u, t have eigenvalues s_n, u_n, t_n , satisfying the sum rule

$$s_n + u_n + t_n - 2m_i^2 - 2M_k^2 = 0.$$

If we now take into account that the processes in the s -channel and the u -channel are physically indistinguishable, and impose on ψ the condition of crossing symmetry

$$\psi(p_i \leftrightarrow p'_i) = \psi,$$

then we obtain $u_n = s_n$, whence

$$s_n - m_i^2 - M_k^2 = -t_n/2. \quad (4)$$

Comparing with (1), we find $x_n = -t_n/2$. Thus, we arrive at the important conclusion that the levels $x(J^P)$ can be interpreted as eigenvalues of the operator of the invariant square of the momentum transfer

$$\tilde{x} = -t/2 = (\vec{p}^2 - p_0^2)/2, \quad p = p_i + p'_i \quad (5)$$

or, more precisely, as its mean values over the eigenfunctions ψ_n . The structure of the spectrum of the levels $x(J^P)$ (Fig. 1 of Ref. ⁽¹⁾) indicates that these mean values turn out to be close to the eigenvalues of the equation

$$\tilde{x}\varphi_n = x_n\varphi_n,$$

corresponding to fairly simple boundary conditions. In order to describe such a picture in potential language, we introduce space-time coordinates ξ , canonically

conjugate to the 4-vector p . We write the equation for determining x_n in the form

$$\{(\vec{p}^2 - p_0^2)/2 + V(\xi)\}\varphi_n(\xi) = x_n\varphi_n(\xi). \quad (6)$$

Of course, the problem of justifying equation (6), which describes the relative motion of two particles, is closely connected with the problem of justifying equation (3) and introducing an interaction into the latter. In the presence of a corresponding crossing-symmetric equation of type (3) for the case of interaction, one may hope to obtain equation (6), for example, by the method of separation of variables under appropriate hypotheses about the structure of the interaction potentials in the initial equation. Let us note that the idea of obtaining the particle mass spectrum from an equation of type (6) is not new. Its realization, however, requires assigning a clear physical meaning to the variables ξ . In this connection a number of difficult problems arise, connected mainly with ensuring the locality of the interaction while preserving the relativistic invariance of the description. Methods for solving these problems were indicated by M. A. Markov⁽⁴⁾, and by D. I. Blokhintsev and G. L. Kolerov⁽⁵⁾.

Returning to equation (6), let us note that the simple structure of the levels $x(J^P)$ corresponds to potentials $V(\xi)$ of the type of a rectangular well or an oscillator (see⁽⁴⁾), the characteristic size of the well being approximately 10^{-14} cm (see⁽¹⁾).

The spin splittings of the $M + B$ levels correspond to a simple spin-unitary spin-orbit coupling

$$V_{MB}(\xi) = V_0(\xi^2) + (s\xi)V_1(\xi^2). \quad (7)$$

Of particular interest is the absence of s -levels in the principal spectral series. One may suppose that the description of the interaction of the systems $M + M$ and $M + B$ by means of two universal potentials $V_{MM}(\xi^2)$ and $V_{MB}(\xi)$ is legitimate only for cases in which the mean distances between hadrons are not too small ($\gtrsim 10^{-14}$ cm). The S -wave states of the principal series correspond to the maximum approach of the particles, at which their internal (for example, quark) structure becomes important and individual localization becomes illegitimate.

Such a hypothesis appears especially plausible in the light of the results of N. N. Bogoliubov⁽⁶⁾, who, within the framework of the constituent-quark model, succeeded in explaining the nucleon $N(940)$ and the Roper resonance $N'(1470)$. In our spectral picture the nucleon drops out noticeably from the level $P(1/2^+)$, while $N'(1470)$ receives no explanation at all. $N'(1470)$, as well as the s -wave resonances of the $M + B$ system $N(1550)$, $N'(1700)$, $\Delta(1640)$, $\Lambda(1405)$, and $\Lambda'(1670)$, apparently should be assigned to the second spectral series.

From this point of view the particles π , K , N are the most compact quark formations. Their intrinsic radii are appreciably smaller than 10^{-14} cm. All

other particles are more diffuse. A study of the upper line of Table 2 from ⁽¹⁾ shows that in the series Λ , Σ , Ξ the degree of diffuseness increases. This apparently corresponds to the decreasing number of resonant states in the series $\pi + \Lambda$, $\pi + \Sigma$, $\pi + \Xi$, observed experimentally.

Note added in proof. The marked dropping out of unitary singlets from the levels $(3/2^-)$ and $(7/2^-)$ may possibly indicate the importance of the symmetry aspect when discussing the nature of the levels $x(J^P)$. In this connection it is of interest to consider the possibility of relating the parameter x not to the masses of the decay products, but to the quark content of the hadrons.

Recently V. L. Chernyak pointed out that if one requires that the parameter y , defined as the square of the hadron mass minus the sum of the squares of the effective masses of the quarks entering into it, be constant within each of the $SU(3)$ multiplets, then the constancy of the parameter x follows as a consequence. Such an approach makes it possible to explain the inclusion of the three-pion resonances ω and A_2 in the levels $P(1^-)$ and $D(2^+)$ of the meson-meson series.

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Note: Figure translations are in progress. See original paper for figures.

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