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Abstract

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NEUTRINO EMISSION OF ENERGY BY STARS WITH ACCOUNT OF THE PAULI PRINCIPLE

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Calculations of the emission of neutrinos and antineutrinos from a star are usually carried out under the assumption that states with a given energy and momentum are free for the creation of the emitted particles ν and $\bar{\nu}$. In most cases this assumption is justified. However, at high temperatures ($T > 10^{10}$ K) and comparatively large sizes of the emitting region of the star, where the formation of neutrino pairs mainly occurs, it becomes necessary to take account of the Pauli principle, i.e., to take account of the degree of occupation of neutrino and antineutrino states.

According to present-day ideas, such conditions may be realized in stars in the course of evolution at the stage of relativistic collapse. According to (1), under adiabatic (loss-free) compression of a star to the density $\bar{\rho} = 2 \cdot 10^{16}(M_{\odot}/M)^2$ g/cm^3, corresponding to gravitational self-closure, the temperature reaches values $T_{10} = 400(M_{\odot}/M)^{1/2}$, where T_{10} is the temperature in units of 10^{10} K, M is the mass of the star, and $M_{\odot}$ is the mass of the Sun.

Intense formation of neutrino pairs with increasing temperature is accompanied by an increase in the energy radiated by neutrinos from the star and, at the same time, by an increase in the concentration of neutrinos in the star. The concentration of neutrinos is proportional to the rate of neutrino formation and to the size of the emitting region. In addition, with increasing temperature and, consequently, with increasing neutrino energy, the interaction of neutrinos with matter increases. When the neutrino concentration constitutes an appreciable fraction of the concentration that is equilibrium at the given temperature, in calculations of neutrino emission it is necessary to take into account the degree of occupation of the states $\nu, \bar{\nu}$. In the limit, as the temperature of the star increases, the neutrino energy emission tends to the equilibrium value corresponding to the state of a neutrino gas in thermodynamic equilibrium with matter.

Below a criterion is obtained for the establishment of thermodynamic equilibrium of neutrinos in a star, approximate formulas are derived for calculating neutrino emission with allowance for occupation numbers, and the upper limit is determined for the masses of collapsing stars in which neutrino equilibrium can be established.

For simplicity we shall assume that in the emitting region of radius R the concentration of neutrinos (antineutrinos) is constant, while the neutrino flux is homogeneous and isotropic. The density and temperature of the matter are constant with radius. In the case of an isotropic flux, the neutrino concentration is related to the flux by the relation

$$dS = c\,dn/4,$$

where dS is the flux of neutrinos with energy E in the interval dE; dn is the number of neutrinos in the interval dE.

The degree of occupation of neutrino states f is defined as the ratio of the neutrino density in the interval dE to the number of quantum states in the phase volume

$$f = dn/d\Gamma, \qquad d\Gamma = E^2 dE/2\pi^2\hbar^3c^3.$$

In a state of thermodynamic equilibrium

$$f = \left(1 + \exp\frac{E-\varphi}{kT}\right)^{-1},$$

φ is the chemical potential of the neutrino. At high temperatures ($kT \gg \varphi_{\text{electron}}$) the chemical potential of the neutrino is close to zero, $\varphi \approx 0$, since under these conditions neutrinos are produced in equal number with antineutrinos and their densities are equal (analogously to the formation of electron-positron pairs ([2])). In the case of electron degeneracy $\varphi \neq 0$. Below, in the calculations, it is assumed that the chemical potential of the neutrino is equal to zero ($kT \gg \varphi_{\text{electron}}$).

The neutrino flux dS in the case of homogeneous and isotropic radiation is related to the rate of neutrino production per unit volume by the relation

$$4\pi R^2\,dS = \frac{4\pi}{3}R^3\,d\nu, \quad \text{i.e.} \quad dS = \frac{R}{3}\,d\nu.$$

The rate of neutrino production is equal to the difference between the rates of birth and absorption of neutrinos,

$$d\nu = d\nu_0(1 - f - \chi f),$$

where $d\nu_0$ is the rate of neutrino production without allowance for the Pauli principle; χ is the ratio of the rates of absorption and birth of neutrinos.

In the case of equilibrium $d\nu = 0$, $f = [1 + \exp(E/kT)]^{-1}$, i.e. $\chi = \exp(E/kT)$. Using these relations, the neutrino flux can be written in the form

$$dS = dS_0[1 - f - f\exp(E/kT)];$$

dS_0 is the neutrino flux without allowance for the degree of filling of states.

Expressing the neutrino density through the flux, we obtain

$$dn = \frac{4}{c}\left(1 - f - f\exp\frac{E}{kT}\right) dS_0.$$

The degree of filling of states is

$$f = \frac{4}{c}\frac{dS_0}{d\Gamma}\left(1 - f - f\exp\frac{E}{kT}\right),$$

i.e.

$$f = \frac{\beta}{1 + \beta + \beta\exp(E/kT)}, \qquad \beta = \frac{4}{c}\frac{dS_0}{d\Gamma}. \tag{1}$$

The neutrino energy flux is equal to

$$I = \int E\,dS = \int \frac{E\,dS_0}{1 + \beta + \beta\exp(E/kT)} = \int \frac{c}{4}E\,\frac{\beta}{1 + \beta + \beta\exp(E/kT)}\,d\Gamma. \tag{2}$$

Under conditions when the neutrino gas is close to thermodynamic equilibrium, $f \sim [1 + \exp(E/kT)]^{-1}$, $\beta \gg 1$. In this case

$$I \approx \int \frac{c}{4}\frac{d\Gamma}{1 + \exp(E/kT)} = \frac{7}{16}aT^4 = I_{\text{equil}};$$

a is the Stefan-Boltzmann constant.

The natural result has been obtained that, in a state close to equilibrium, the emission of neutrino energy has an equilibrium character. The appearance of the factor 7/16 in comparison with photon radiation is connected with the fact that the neutrino obeys Fermi-Dirac statistics and, in addition, in a state with a given momentum only one value of the polarization is possible. For the antineutrino, analogously, $I_{\text{equil}} = {}^7/_{16}aT^4$.

When the degrees of filling of neutrino states are small, $f \sim 0$, $\beta \sim 0$, $I = \int E\,dS_0$, i.e. the approximation is obtained in which the neutrino emissivity is usually calculated.

In the case $f \ll 1$, $\beta \ll 1$, to accuracy up to the first term of the expansion:

$$I \approx \int E\left(1-\beta-\beta\exp\frac{E}{kT}\right)dS_0 = \int E\left[1-\frac{8\pi^2\hbar^3c^2[1+\exp(E/kT)]}{E^2}\frac{dS_0}{dE}\right]dS_0. \tag{3}$$

As will be shown below, the mean free path of a neutrino at $f \ll 1$ is much larger than the dimensions of a star, $l \gg R$, and the neutrino radiation has a volumetric charac-

Therefore, instead of the neutrino energy flux one can introduce the losses of neutrino energy per unit volume per unit time q:

$$q \approx \int E\left[1-\frac{8\pi^2\hbar^3c^3[1+\exp(E/kT)]R}{3E^2}\frac{d\nu_0}{dE}\right]d\nu_0. \tag{4}$$

Using formula (4), let us consider neutrino emission with allowance for the Pauli principle in specific examples (analogous results are obtained for antineutrinos).

As is known, at high temperatures ($kT \gg mc^2$, m is the electron mass) the basic processes of formation of neutrino pairs are annihilation of electron–positron pairs $e^+ + e^- \to \nu + \bar{\nu}$ (3) (if there is electron-neutrino interaction $(\bar{e}\nu)(\bar{\nu}e)$) and the interaction of electrons and positrons with nuclei (4), in particular, with free nucleons (5). In the latter case, the process includes three reactions:

$$\mathrm{p} + e^- \to \mathrm{n} + \nu, \qquad \mathrm{n} \to \mathrm{p} + e^- + \bar{\nu}, \qquad \mathrm{n} + e^+ \to \mathrm{p} + \bar{\nu}.$$

The β-decay of the neutron at high temperatures is considerably slower than electron and positron capture, and it may be neglected.

We find the rate of neutrino formation $d\nu_0$, assuming $kT \gg mc^2$, $kT \gg E_0$, $kT \gg \varphi$; E_0 is the energy of neutron β-decay; φ is the chemical potential of the electrons. According to (5), in this approximation the energy of the neutrino produced is equal to the electron energy $E_\nu \approx E_e$;

$$d\nu_0 = \sigma v N_\mathrm{p} dn_e, \qquad \text{where} \quad dn_e = E^2 dE/\pi^2\hbar^3c^2[1+\exp(E/kT)];$$

N_p is the number of protons per unit volume; v is the relative velocity of the electron.

Using the expression for β, it is easy to obtain the criterion for the occupation numbers of neutrino states:

$$f \sim [1+\exp(E/kT)]^{-1}, \qquad \text{when} \quad \beta \gg 1 \quad \text{or} \quad \frac{8}{3}\frac{R}{c}\sigma v N_{\rm p} \gg 1,$$

i.e. $R/c \gg \tau$, where τ is the characteristic time of neutrino formation. At high temperatures the rates of the reactions $\mathrm{p} + e^- \to \mathrm{n} + \nu$, $\mathrm{n} + \nu \to \mathrm{p} + e^-$ are close and $N_{\rm p} \approx N_{\rm n}$ (6); therefore the condition $R/c \gg \tau$ may be written in the form $R \gg l$, l being the mean free path. Thus, equilibrium of the neutrino with matter is established when the time of formation (and absorption, since $\tau \approx \tau_{\rm abs}$) of the neutrino is considerably less than the time of flight of the neutrino through the star.

The occupation numbers of the states are small under the condition $\tau \gg R/c$, $l \gg R$. Using the cross section σ and the concentration of protons from (5), we obtain

$$d\nu_0 = \frac{\ln 2}{2tf}\frac{N}{(mc^2)^5}\frac{E^4\,dE}{1+\exp(E/kT)};$$

$$q_\nu \approx q_{0\nu}\left[1 - \frac{4}{3}\pi^2 \ln 2\,\frac{RN}{tf}\frac{\hbar^3 (kT)^2}{m_e^5 c^8}\frac{J_1}{J_2}\right];$$

$$J_1 = \int_0^\infty \frac{x^7\,dx}{1+e^x} = \frac{127}{240}\pi^8; \qquad J_2 = \int_0^\infty \frac{x^5\,dx}{1+e^x} = \frac{31}{252}\pi^6; \qquad x = \frac{E}{kT};$$

$$q_{0\nu} = 0.2\cdot 10^{20}\frac{\rho}{tf}\left(\frac{kT}{mc^2}\right)^6 \frac{\text{erg}}{\text{cm}^3\cdot\text{sec}};$$

tf is the comparative neutron half-life period, $tf = 1190$ sec.; N is the number of nuclei per unit volume, $N = 6\cdot 10^{23}\rho$.

Substitution of numerical coefficients gives

$$q_\nu \approx q_{0\nu}\,(1 - 10^{-18} R\rho T_{10}^2)\,,$$

where the units of measurement are R in cm, ρ in g/cm^3.

Thus, the neutrino is close to thermal equilibrium when $\varphi T_{10}^2 R \sim 10^{18}$, i.e., for a star with mass $M \sim 2\cdot 10^{21}\rho^{-2}T_{10}^{-6}M_\odot$. An analogous estimate, accurate to within a factor, is obtained from the condition that ${}^4/{}_3\pi R^3 q_{0\nu} \sim 4\pi R^2 I_{\rm eq}$.

In a collapsing star $T_{10} < 400\,(M_\odot/M)^{1/2}$, $\rho < 2\cdot 10^{16}(M_\odot/M)^2$. Substituting these expressions into the formula for the stellar mass, we find that, in principle, establishment of equilibrium of neutrinos with matter during collapse is possible in stars with mass $M < 3\cdot 10^4 M_\odot$. The upper mass limit is connected with the fact that, during the collapse of massive stars ($> 3 \cdot 10^4 M_\odot$), the temperature and density are comparatively low for the neutrino formation time to be much less than the time of its escape from the star.

In the case of the process $e^+ + e^- \to \nu + \bar{\nu}$ (electron-neutrino interaction), in the approximation $kT \gg mc^2$, $kT \gg \varphi$, in an analogous way, using the expression for the cross section and the concentrations of electrons and positrons from (3), we obtain

$$q_\nu \approx q_{0\nu} \left[1 - \frac{280}{135} \pi^2 \sigma_0 R \left(\frac{mc}{\hbar} \right)^3 \left(\frac{kT}{mc^2} \right)^5 \right] = q_{0\nu} \left(1 - 7.4 \cdot 10^{-12} R T_{10}^5 \right),$$

$$\sigma_0 = 1.5 \cdot 10^{-45} \text{ cm}^2, \qquad q_{0\nu} \approx 2.2 \cdot 10^{24} T_{10}^9 \text{ erg/cm}^3 \cdot \text{sec}.$$

The neutrino gas is close to equilibrium when $RT_{10}^5 \sim 1.4 \cdot 10^{11}$, or $M \sim 5.7 \rho T_{10}^{-15} M_\odot$. Using the fact that $T_{10} < 400 (M_\odot / M)^{1/2}$, we find the upper mass limit for a collapsing star in which thermodynamic equilibrium of neutrinos with matter can be established through the electron-neutrino interaction: $M < 8 \cdot 10^5 \rho^{-2/13} M_\odot$.

The formulas obtained above are approximate in character. In the calculations, homogeneity of the neutrino density was assumed, as well as homogeneity and isotropy of the flux. In reality, in a star, owing to boundary effects and also to the distribution of temperature and density with radius, these assumptions are not justified.

An exact approach to the problem requires consideration of the kinetic equation. However, taking into account the spatial and angular distribution of neutrinos, the density and temperature profiles, boundary effects, etc., encounters great mathematical difficulties. The idealization of the problem adopted above, without changing the result in order of magnitude, makes it possible to establish the basic regularities comparatively simply.

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Note: Figure translations are in progress. See original paper for figures.