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VELOCITY FIELD EXCITED BY WING VIBRATIONS PROPAGATING OVER THE SURFACE AT SUPERSONIC SPEED

AERODYNAMICS

1968

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

UDC 533.6

AERODYNAMICS

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VELOCITY FIELD EXCITED BY WING VIBRATIONS PROPAGATING OVER THE SURFACE AT SUPERSONIC SPEED

(Presented by Academician L. I. Sedov, 12 II 1968)

1. Let us consider the rectilinear translational motion of a wing with constant velocity u within an unbounded volume of an ideal compressible medium. Beginning at some instant of time t_0, small oscillations propagate over the elastic surface of the wing with supersonic speed v. The normal component of the velocity due to the basic motion is prescribed on both sides of the wing in the form

$$v_{0n} = -u\alpha, \tag{1}$$

where α is the angle of attack of the elements of the surface being flowed around. The normal component of the velocity due to the vibrations is prescribed in the form

$$v_{\Delta n} = A_\Delta, \tag{2}$$

where A_Δ is a function of time and of points of the surface being flowed around. The functions α and A_Δ are small and may be arbitrary integrable functions of their arguments.

Fig. 1

Assuming that the medium is weakly disturbed, we consider the problem of determining the velocity field in a linearized formulation ([1,2]). We shall suppose the motion of the medium to be irrotational and to occur in the absence of external forces. We take a moving coordinate system $Oxyz$, rigidly attached to

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Fig. 2

Figure 2: Fig. 2

the moving wing (Fig. 1). The axis Oz is directed perpendicular to the plane of the figure.

The velocity potential φ satisfies the equation

$$(u^2 - a^2)\varphi_{xx} - a^2\varphi_{yy} - a^2\varphi_{zz} - 2u\varphi_{xt} + \varphi_{tt} = 0, \tag{3}$$

where a is the speed of sound in the undisturbed medium, and the boundary conditions are in the plane xOy.

In the region Σ_0—the projection of the wing onto the plane xOy ahead of the front of vibration propagation (the line FF_1 in Fig. 1)—the derivative is

$$\varphi_z = -u\alpha(x, y) = A_0(x, y). \tag{4}$$

In the region Σ—the projection of the wing onto the plane xOy behind the front FF_1—the derivative is

$$\varphi_z = A_0(x, y) + A_\Delta(x, y, t) = A(x, y, t). \tag{5}$$

In the region Σ_1—the projection of the vortex wake onto the plane xOy,

$$\varphi_t - u\varphi_x = 0. \tag{6}$$

Everywhere in the plane xOy, outside the region $\Sigma_0 + \Sigma + \Sigma_1$, the potential is

$$\varphi = 0. \tag{7}$$

In addition, at each instant of time the Chaplygin–Zhukovsky principle must be satisfied at the trailing edge of the wing.

Thus, the problem is as follows. Find, in the half-space $z \geqslant 0$, a function $\varphi(x, y, z, t)$ that satisfies equation (3), the boundary conditions (4)–(5), specified in regions with a moving boundary, and the boundary conditions (6)–(7), specified in regions with fixed boundaries. The solution of the problem in the half-space $z < 0$ is found from the condition

$$\varphi(x, y, -z, t) = -\varphi(x, y, z, t).$$

Fig. 2

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2. To solve the problem we shall apply the method developed in the papers ([3],[4]). Let us turn to the space xyt and consider in it a region V, inside which the derivatives φ_z are specified by the flow condition. The region V is bounded by the surface Σ^*. The surface Σ^* is a cylindrical surface with generators parallel to the time axis Ot, and with directrix given by the contour $AOBD$, which is specified by the equation: $\eta = \psi(\xi)$ (Fig. 2).

Let the line FF_1 be the front of propagation of vibrations along the wing surface, moving according to the law: $x = f(t)$, where $f'(t) = v$. The surface F^*, specified by the equation $\xi = f(\tau)$, divides the region V into two parts V_1 and V_2 with different values of the derivative φ_z, according to conditions (4) and (5). To the left of the surface F^*, in the region V_1, the derivative $\varphi_z = A_0$; to the right—in the region V_2, the derivative $\varphi_z = A$ (Fig. 2).

We take the solution of equation (3) in the form ([3])

$$\varphi(x,y,z,t) = \frac{u^2-a^2}{2\pi} \iint_{S(x,y,z,t)} \frac{\varphi_z\left(\xi,\eta,0,t-\dfrac{u(x-\xi)+ar}{u^2-a^2}\right) dS}{\sqrt{(u^2-a^2)^2r^2+[a(x-\xi)-ur]^2+a^2k^4(y-\eta)^2}}, \tag{8}$$

$$r = \sqrt{(x-\xi)^2-k^2(y-\eta)^2-k^2z^2}, \qquad k=\sqrt{u^2/a^2-1},$$

where the region of integration S is the surface of a hyperboloid determined by the equation

$$(x-\xi)^2+(y-\eta)^2+z^2+2u(x-\xi)(t-\tau)+(u^2-a^2)(t-\tau)^2 = 0 \tag{9}$$

and the inequality $\tau < t$. In formula (8) we pass from the surface integral to double integrals with a plane region of integration in the plane xOy. Let us consider the supersonic velocity of wing motion $u > a$; then we obtain

$$\varphi(x,y,z,t) = -\frac{1}{2\pi}\iint_{S^*(x,y,z)} \frac{\varphi_z(\xi,\eta,0,\tau_1)}{r}\, d\xi d\eta - \frac{1}{2\pi}\iint_{S^*(x,y,z)} \frac{\varphi_z(\xi,\eta,0,\tau_2)}{r}\, d\xi d\eta. \tag{10}$$

$$\tau_1 = t+\frac{u(x-\xi)+ar}{u^2-a^2}, \qquad \tau_2 = t+\frac{u(x-\xi)-ar}{u^2-a^2}.$$

The region S^* is bounded above by the Mach wave, and below by the Mach hyperbola or, for $z=0$, by the Mach lines.

In constructing the solution, an essential role is played by the line of intersection of the surfaces S and F^*. Let us denote the projection of this line onto the plane xOy by l. The curve l divides the plane region S^* into parts with different values of the derivative φ_z. If the front of propagation of the vibrations over the wing surface is a straight line FF_1, moving opposite to the motion of the wing with constant velocity

Fig. 3

Fig. 4

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Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

$v > u + a$, then the surface F^* is a plane defined by the equation $\xi + v\tau = 0$, and the curve l is an ellipse defined by the equation

$$v^2(x-\xi)^2 + v^2(y-\eta)^2 + v^2z^2 +$$

$$+2uv(x-\xi)(vt+\xi) + (u^2-a^2)(vt+\xi)^2 = 0. \quad (11)$$

We note that the ellipse l is always inscribed in the Mach hyperbola or, for $z = 0$, in the angle formed by the Mach lines.

Assuming that on the contour $AOBD$ the condition

$$\left| u\psi'(\xi)/\sqrt{\psi i'^2(\xi)+1} \right| \geq a,$$

is satisfied, we represent the solution (10) in the form

$$\varphi(x,y,z,t) = \varphi_0(x,y,z) - \quad (12)$$

$$-\frac{1}{2\pi}\iint_{\sigma(x,y,z,t)+\sigma_1(x,y,z,t)} \frac{A_\Delta(\xi,\eta,\tau_1)}{r}\,d\xi d\eta - \frac{1}{2\pi}\iint_{\sigma_1(x,y,z,t)} \frac{A_\Delta(\xi,\eta,\tau_2)}{r}\,d\xi d\eta,$$

where the region of integration σ is the part of the region S^* that, at the time t, lies inside the ellipse l, while the region σ_1 is the part of the region S^* lying outside the ellipse l, below the arc of the ellipse K_1LK_2. The points K_1 and K_2 are the points of tangency of the ellipse l with the Mach hyperbola (Fig. 3). The function φ_0 in formula (12) is the solution of the known problem on the flow past the wing under consideration by a steady supersonic gas flow ([5]).

3. The analytical form of the solution of the problem depends on the relative position of the ellipse l and the wing contour $AOBD$, which determines the regions of integration σ and σ_1 in the solution (12).

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Consider a time t belonging to the interval $0 = t_0 \leq t \leq t_1 = d/v$, where d is the length of the segment OD. In the space xyt we draw the plane $\tau = t$. The projection of the line of intersection of the plane $\tau = t$ with the plane-

by the surface F^* on the plane xOy, and denote it by FF_1 (Fig. 2). Let us consider in the space xyt a family of cones defined by the equation

$$(X-\xi)^2 + (Y-\eta)^2 + 2u(X-\xi)(T-\tau) + (u^2-a^2)(T-\tau)^2 = 0$$

and by the inequality $\tau > T$, with vertices on the line of intersection of the plane F^* and the surface Σ^*. Denote the envelope surface of this family by Ω. Let the projections of the lines of intersection of the plane $\tau = t$ with the envelope Ω onto the plane xOy be denoted by Ω_1 and Ω_2. We find the equations of the curves Ω_1 and Ω_2 in parametric form

$$v^2(x^*-\xi)^2 + v^2[\psi(x^*)-\eta]^2 - 2uv(x^*-\xi)(x^*+vt) +$$

$$+(u^2-a^2)(x^*+vt)^2 = 0,$$

$$v^2(x^*-\xi) + v^2[\psi(x^*)-\eta]\psi'(x^*) - uv(x^*-\xi) - \tag{13}$$

$$uv(x^*+vt) + (u^2-a^2)(x^*+vt) = 0,$$

where x^* is the parameter.

The lines FF_1, Ω_1, Ω_2 separate regions with different analytic character of the solution of the problem. To the region enclosed between the straight line FF_1 and the curve Ω_1 there corresponds the solution (12), in which the region of integration σ is the part S^* bounded by the ellipse l, lying entirely inside S^* (Fig. 2). To the region enclosed between the curves Ω_1 and Ω_2 there corresponds the solution (12), in which σ is the part of S^* cut off by the ellipse l, which intersects the wing contour. In this case the points of tangency K_1 and K_2 may lie both inside the wing (Fig. 3) and outside it (Fig. 4). To the region enclosed between the curve Ω_2 and the Mach wave there corresponds the solution (12), when in both integrals the integration extends over the whole region S^*, i.e., the region σ is absent, and the region σ_1 coincides with the region S^*.

If the velocity of propagation of the vibrations along the surface in the flow satisfies the inequality $v < u + a$, then the curve l is a hyperbola.

4. In the particular case when harmonic oscillations with frequency ω propagate along the elastic surface of the wing, the function

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$$A_\Delta(x,y,t) = A_1(x,y)\exp i\omega(t+\alpha_1(x,y)) = \operatorname{Re} A_2(x,y)\exp i\omega t.$$

Using the relation

$$\exp\frac{i\omega a}{u^2-a^2}r + \exp\frac{-i\omega a}{u^2-a^2}r = 2\cos\frac{\omega a}{u^2-a^2}r,$$

we write solution (12) in the form

$$\varphi = \varphi_0 - \frac{1}{2\pi}\operatorname{Re}\exp(i\omega t)\exp\left(\frac{i\omega u}{u^2-a^2}x\right)\times$$

$$\times\iint_\sigma \frac{A_2(\xi,\eta)}{r}\exp(i\omega t)\exp\left(\frac{-i\omega u}{u^2-a^2}\xi\right)\exp\left(\frac{i\omega a}{u^2-a^2}r\right)d\xi d\eta -$$

$$-\frac{1}{2\pi}\operatorname{Re}\exp(i\omega t)\exp\left(\frac{i\omega u}{u^2-a^2}x\right)\times$$

$$\times\iint_{\sigma_1}\frac{A_2(\xi,\eta)}{r}\exp\left(-\frac{i\omega u}{u^2-a^2}\xi\right)\cos\frac{\omega a}{u^2-a^2}r\,d\xi d\eta,$$

where the prescribed function A_2 determines the amplitude and initial phase of the oscillations at each oscillating point on the wing surface.

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Received
10 I 1968

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