ON ONE METHOD FOR THE SYNTHESIS OF A NOISE-SUBSTITUTED SYSTEM WITH VARIABLE STRUCTURE

K. K. ZHILTSOV, P. A. ILOKHIN, A. V. LANYSHIN

1968

SovietRxiv

Abstract

Full Text

UDC 62.503.2

CYBERNETICS AND CONTROL THEORY

K. K. ZHILTSOV, P. A. ILOKHIN, A. V. LANYSHIN

ON ONE METHOD FOR THE SYNTHESIS OF A NOISE-SUBSTITUTED SYSTEM WITH VARIABLE STRUCTURE

(Presented by Academician B. N. Petrov on 5 X 1967)

The possibility is considered of constructing a system with variable structure in the presence of noise, specified in the form of a random stationary process with a normal distribution law, which acts on the logic-forming block.

In the absence of noise, the motion of the system with variable structure is described by the system of differential equations (1)

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \overline{\psi});$$

$$\mathbf{x} = (x_1, \dots, x_n); \qquad \overline{\psi} = (\psi_1, \dots, \psi_{n-1}); \qquad \mathbf{f} = (f_1, \dots, f_n);$$

$$f_i = x_{i+1} \ (i = 1, 2, \dots, n-1); \qquad f_n = -\sum_{i=1}^{n} a_i x_i - \sum_{i=1}^{n-1} \psi_i(\mathbf{x}) x_i; \tag{1}$$

$$\psi_i(\mathbf{x}) = \begin{cases} \omega_i, & g x_i > 0, \\ \lambda_i, & g x_i < 0, \end{cases} \qquad (i = 1, 2, \dots, n-1);$$

$$g = \sum_{i=1}^{n} c_i x_i; \tag{2}$$

ω_i, λ_i, c_i are constant quantities; $c_n = 1$; a_i are time-varying coefficients.

After the representative point reaches the hyperplane $g = 0$, under satisfaction of the conditions of the sliding mode, the motion of the system is described by the equation

$$\sum_{i=1}^{n} c_i x_i = 0. \tag{3}$$

When additive random noise acts through the signal channels x_i $(i = 1, \dots, n)$ on the logic-forming block, the conditions for existence of the sliding mode are violated, as a result of which the character of the system motion changes. It can be approximately described by a system of nonlinear differential equations written for mathematical expectations. These equations are obtained on the basis of the method of statistical linearization (2), taking into account complete filtering of the random component along the system loop

$$d\mathbf{m}/dt = \mathbf{f}(\mathbf{m}, \mathbf{m}_\psi),$$

$$\mathbf{m} = (m_1, \dots, m_n); \qquad \mathbf{m}_\psi = (m_{\psi 1}, \dots, m_{\psi(n-1)}); \qquad \mathbf{f} = (f_1, \dots, f_n);$$

$$f_i = m_{i+1} \ (i = 1, 2, \dots, n-1); \qquad f_n = -\sum_{i=1}^{n} a_i m_i - \sum_{i=1}^{n-1} m_{\psi i}(\mathbf{m}) m_i;$$

$$m_{\psi i} = \frac{m_i^-}{2}\left[4\Phi\left(\frac{m_i}{\sigma_i}\right)\Phi\left(\frac{m_g}{\sigma_g}\right) + \Phi\left(\frac{m_g}{\sigma_g}\right)\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(m_i/\sigma_i)^2} + \rho_{ig}\Phi\left(\frac{m_i}{\sigma_i}\right)\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(m_g/\sigma_g)^2}\right] + \frac{m_i^+}{2}; \tag{4}$$

$$m_i^+ = m_i(\omega_i + \lambda_i); \qquad m_i^- = m_i(\omega_i - \lambda_i);$$

$m_i, \ m_{\psi i}, \ m_g$ are the mathematical expectations of the signals $x_i, \ \psi_i, \ g$; ρ_{ig} is the coefficient of mutual correlation between x_i and g; $\sigma_i^2, \ \sigma_g^2$ are the variances of the noise acting in the channels x_i and g,

$$\Phi(y) = \frac{1}{\sqrt{2\pi}} \int_0^y e^{-t^2/2}\, dt.$$

For

$$|m_i/\sigma_i| \to \infty, \qquad |m_g/\sigma_g| \to \infty$$

(4) degenerates into (1).

Fig. 1

Figure 1: Fig. 1

The paper considers the practically most important case, when

$$|m_i/\sigma_i| \to \infty, \qquad |m_g/\sigma_g| \text{ is small}$$

Analysis shows that in this case, in the automatic control system described by (4), self-oscillations may arise, and the amplitude of the self-oscillations increases with increasing σ_g^2.

Fig. 1

Effective filtering of noise acting in the channels x_i and g, using linear elements with a transfer function of the form

$$W_1(p) = M(p)/N(p), \tag{5}$$

where

$$M(p) = \sum_{i=1}^{m} b_i p^{i-1}, \qquad N(p) = \sum_{i=1}^{q} d_i p^{i-1}, \qquad q - m - n \ll 1, \tag{6}$$

leads to inadmissible phase lags in the signals x_i and g. In the present paper it is proposed to reduce the noise level at the input of the nonlinear logic device by synthesizing variable-structure systems in accordance with the structural diagram of Fig. 1.

In this case the phase lag in the signals x_i, g is compensated by means of a lead element with transfer function $W_2(p) = 1/W_1(p)$ in the forward path. It is not difficult to see that, in the space of the variables y_i in Fig. 1, in the absence of disturbances $z(t) = 0$, both the sliding-mode conditions and the equation of motion in the sliding mode coincide with (3). In the presence of disturbances, their level, by choosing the structure and the parameters of the filter $W_1(p)$, can be reduced to a small value, which ensures motion close to (3).

Then the equation of motion in the sliding mode with respect to the variables x_i is written in the form

$$\sum_{i=1}^{k} \xi_i x_i = 0, \qquad k = (n-1)(q-1) + 1, \tag{7}$$

where

$$\xi_i = \sum_{k=1}^{i} b_k d_{i-k+1}.$$

In the special case $M(p) = b_1$, the equations of motion in the spaces x_i and y_i in the sliding mode coincide.

Institute of Automation and Telemechanics
(Technical Cybernetics)

Received
28 IX 1967

REFERENCES

[1] S. V. Emel' yanov, V. I. Utkin, DAN, 152, No. 2 (1963). [2] I. E. Kazakov, B. G. Dostupov, *Statistical Dynamics of Nonlinear Automatic Systems*, 1964.

Note: Figure translations are in progress. See original paper for figures.