ACTION OF AN ACOUSTIC PRESSURE WAVE ON A SHALLOW SPHERICAL SHELL

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Abstract

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ACTION OF AN ACOUSTIC PRESSURE WAVE ON A SHALLOW SPHERICAL SHELL

The interaction of a plane shock pressure wave with an elastic shallow spherical shell immersed in an ideal compressible fluid is considered. Along its circular contour the shell is fastened by means of an elastic rib to an absolutely rigid immovable spherical screen. The wave front is parallel to the plane of the supporting contour.

The linear equations of axisymmetric motion of a shallow spherical shell closed at the vertex may be represented in the form ([1])

$$\frac{D}{R_0^2}\nabla^2\nabla^2 w_1 - N_0\nabla^2 w_1 + \tag{1}$$

$$+\frac{Eh}{1-\mu}\left[(1-\mu)\varphi_0^2 w_1 + 2(1+\mu)\varphi_0^2\int_0^1 w_1\alpha\,d\alpha - 2\varphi_0 V_0\right] + \rho_0 h R_0^2\frac{\partial^2 w_1}{\partial t^2} = R_0^2 p,$$

where

$$\alpha = \frac{\varphi}{\varphi_0}, \qquad \nabla^2 = \frac{\partial^2}{\partial\alpha^2} + \frac{1}{\alpha}\frac{\partial}{\partial\alpha}, \qquad D = \frac{Eh^3}{12(1-\mu^2)}.$$

Here $w_1(\alpha, t)$ is the deflection of the shell in the direction of the inward normal; φ_0, R_0 are the semivertex angle and the radius of the panel base; φ is the angle measured from the vertex of the shell; μ, E, ρ_0 are, respectively, Poisson' s ratio, the modulus of elasticity, and the density of the shell material; h is the shell thickness; V_0 is the displacement of the supporting contour in the tangential direction; N_0 is the initial constant force in the shell; p is the total hydrodynamic pressure acting on the shell; t is time.

The boundary conditions at $\alpha = 1$ will be

$$w_1 = 0, \qquad \frac{D}{R_0^2}\left(\frac{\partial^2 w_1}{\partial \alpha^2} + \frac{\mu}{\alpha}\frac{\partial w_1}{\partial \alpha}\right) + \frac{C_1}{R_0}\frac{\partial w_1}{\partial \alpha} = 0, \tag{2}$$

$$\frac{Eh}{1-\mu}\left(V_0 - 2\varphi_0 \int_0^1 w_1 \alpha \, d\alpha\right) + \frac{EF}{R_0} V_0 = 0,$$

where EF, C_1 are the tensile and torsional stiffnesses of the rib.

The hydrodynamic pressure p acting on the shell will be determined approximately on the basis of the work ([2]) (without taking diffraction from the ribs into account). Since in what follows a very shallow panel will be considered ($\varphi_0 = \pi/12$), it may be assumed that the pressure due to the incident and reflected waves does not depend on the angle φ, and that the character of its variation in time is the same as for $\varphi = 0$, i.e., we assume that the incident wave immediately encompasses the entire panel. Then, under the action of a step impulse, we obtain

$$p = p_0 \left[1 + e^{-ct/R} \cos\left(\frac{ct}{R}\right)\right] H(t) - -$$

$$-\rho c \int_0^t \dot{w}_1 e^{-\frac{c}{R}(t-t_1)} \cos\left[\frac{c}{R}(t-t_1)\right] dt_1, \tag{3}$$

where p_0 is the pressure at the front of the incident wave; ρ, c are, respectively, the density of the medium and the speed of sound in it; R is the radius of the middle surface of the shell ($R_0 \simeq R\varphi_0$); $H(t)$ is the unit Heaviside function.

Let us consider the particular case in which the frame stiffnesses satisfy the relations

$$EF = EhR_0/(1+\mu), \qquad C_1 = Eh^3/12(1+\mu)R_0. \tag{4}$$

In this case equation (1) and the boundary conditions (2) are simplified. After eliminating V_0 by means of the last condition (2) and substituting (3) into equation (1), this equation and the first two boundary conditions (2), taking (4) into account, take the form

$$a\nabla^2\nabla^2 w + b\nabla^2 w + nw + \beta_0 k^2 \ddot{w} =$$

$$= P_0(1 + e^{-\tau}\cos\tau)H(\tau) - \beta k^2 \int_0^\tau \ddot{w} e^{-(\tau-\tau_1)} \cos(\tau - \tau_1)\, d\tau_1, \tag{5}$$

$$w = \nabla^2 w = 0 \qquad \text{at } \alpha = 1.$$

Fig. 1

Figure 1: Fig. 1

Here

$$w = w_1/h, \qquad a = h^4/12(1-\mu^2)R_0^4, \qquad b = N_0 h/ER_0^2, \qquad n = h^2\varphi_0^2/R_0^2,$$

$$k = h/R, \qquad \beta = \rho c^2/E, \qquad \beta_0 = \rho_0 c^2/E, \qquad P_0 = p_0/E, \qquad \tau = ct/R. \tag{6}$$

Apply to equation (5) the Laplace integral transform with respect to the independent variable τ, taking into account the zero initial conditions $w(\alpha, 0) = \dot{w}(\alpha, 0) = 0$, and also the Hankel transform with finite limits with respect to the variable α. Then we shall have

$$\overline{W}^*(\xi_i, s) = A(s)/B_i(\xi_i, s),$$

$$A(s) = (1+2s)[1+(1+s)^2], \tag{7}$$

$$B_i(\xi_i, s) = s(s+1)[(s^2+\gamma_i)\times$$

$$\times(s^2+2s+2)+\eta s^2(s+1)],$$

where

$$W = w\beta_0 k^2/P_0, \qquad \eta = \beta/\beta_0 k,$$

$$\gamma_i = (a\xi_i^4 + b\xi_i^2 + n)/\beta_0 k^2, \tag{8}$$

where the values ξ_i are determined from the equation $J_0(\xi_i) = 0$ (J_0 is the Bessel function of zero order); the bar denotes the Laplace transform and the asterisk the Hankel transform.

Fig. 1

Carrying out the inverse Laplace transform for fractional-rational functions and applying the inversion formula for the Hankel transform with finite limits, we obtain the exact solution of the problem within the adopted assumptions

$$W(\alpha,\tau) = 2\sum_i \frac{J_0(\xi_i\alpha)}{\xi_i J_1(\xi_i)}\, Q_i(\tau),$$

$$Q_i(\tau) = \sum \frac{A(s_j)}{B_i'(s_j)} e^{s_j\tau} + 2\ \mathrm{Re} \sum \frac{A(s_j)}{B_i'(s_j)} e^{s_j\tau}, \qquad (9)$$

where the first sum in $Q_i(\tau)$ extends over all real roots s_j of the function $B_i(s,\xi_i)$, and the second over all complex roots s_j with positive imaginary parts.

Consider the action of a step pressure impulse on a duralumin panel in water: $k = 0.01$; $\varphi_0 = \pi/12$; $\beta_0 = 7.82\cdot 10^{-2}$; $\beta = 3.01$.

$\cdot 10^{-2}$. Figure 1 presents the time dependences of the deflection W (curve 1), velocity $\dot{W}$ (curve 2), acceleration $\ddot{W}$ (curve 3), and meridional force $N_1(T)$ (curve 4) at the center of the panel ($b = 0$). In the same figure, curve 5 characterizes the change in the force $N_1(T)$ for $\alpha = 1$, curve 6 corresponds to the change in the elastic moment $M_0(M)$ at the support, and curve 7 shows the displacement of the support contour $V_0(V)$

$$T = N_1 k\beta_0/EhP_0, \qquad M = M_0 R_0 k^2 \beta_0/C_1 h P_0,$$

$$V = V_0 k^2 \beta_0/(1+\mu)h\varphi_0 P_0. \qquad (10)$$

The solution in the form (9) has good convergence, and in practice, to obtain a prescribed accuracy, one may restrict consideration to a small number of terms of the series.

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Note: Figure translations are in progress. See original paper for figures.