PARAMETRIC INSTABILITY OF THE SURFACE OF A LIQUID IN A VARIABLE ELECTRIC FIELD

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PARAMETRIC INSTABILITY OF THE SURFACE OF A LIQUID IN A VARIABLE ELECTRIC FIELD

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The natural frequency of gravitational-capillary waves on the surface of a liquid depends, in an electric field, on its intensity. In a variable field the natural frequency turns out to be a periodically varying parameter. Therefore, in a variable electric field, parametric excitation of surface instability and the occurrence of parametric waves are possible. Owing to energy dissipation there must exist a threshold of parametric instability. The threshold values of the field intensity can be found within the framework of the linear theory of stability.

1. We consider the stability of the equilibrium of a horizontal liquid surface in a homogeneous electric field $\mathbf{E} = E_0 \cos \omega t$, normal to the surface.

It is assumed that the electric field does not penetrate into the liquid; this corresponds either to an ideally conducting liquid or to a liquid with a very large dielectric permittivity.

The disturbances are assumed to be periodic in the horizontal plane. The equations for the disturbances are obtained in the usual way from the system of equations for a viscous incompressible liquid and Laplace' s equation for the electric potential. The electric and hydrodynamic quantities enter different equations and are connected with one another only by the boundary conditions on the liquid surface. The boundary conditions contain a dependence on time through the equilibrium variable field.

Integrating the equations for the disturbances with respect to the coordinates, we obtain from the boundary conditions

$$\ddot{\xi} + 4\frac{\eta}{\rho}k^2\dot{\xi} + \left[\Omega^2 - \frac{k^2}{8\pi\rho}E_0^2 \cos 2\omega t\right]\xi = 0. \quad (1)$$

This equation determines the time dependence of the displacement of the surface ξ. Here $\Omega^2 = \frac{\alpha}{\rho}k^3 + gk - \frac{k^2 E_0^2}{8\pi\rho}$ is the time-averaged value of the variable

Fig. 1

Figure 1: Fig. 1

parameter of the problem; k is the wave number, η is the viscosity, ρ is the density, α is the coefficient of surface tension of the liquid, and g is the acceleration of free fall.

A special feature of the problem is the dependence of the mean value of the natural frequency on the amplitude $k^2 E_0^2/8\pi\rho$ of the modulation of the parameter. Owing to the quadratic dependence of the force on the field intensity, the modulation frequency is equal to twice the frequency of variation of the field.

The boundaries of the resonance regions of instability of the solutions of equation (1) are constructed by the method of expansion in the modulation amplitude, which is assumed small compared with Ω^2. The smallest critical value of the intensity is

$$E_1 = 8\sqrt{\pi\eta\omega}. \qquad (2)$$

is obtained for $\Omega = \omega$, i.e., when the natural frequency coincides with half the modulation frequency. Since the spectrum of wave numbers and natural frequencies for an unbounded liquid surface is continuous, the latter condition is satisfied at any excitation frequency and determines the wave number of the perturbation that destroys stability. In view of the assumption that the modulation amplitude is small, the applicability of formula (2) is limited to frequencies satisfying the relation

$$\omega \gg 4\frac{\eta}{\rho}k^2.$$

Let us note that in a variable electric field parametric instability may arise not only at the free surface of a conducting liquid, but also at conductor-dielectric and dielectric-dielectric interfaces.

Fig. 1

In addition to the instability mechanism considered above, a nonparametric (static) mechanism is possible, for which the threshold voltage E_2 is determined from the condition $\Omega = 0$ and does not depend on ω. Since the threshold of parametric instability E_1 increases with ω (formula (2)), at sufficiently high frequencies $E_2 < E_1$, and the static mechanism of instability is realized. This type of instability has been investigated both in constant [1] and in variable [2, 3] fields.

2. Parametric excitation of waves in a liquid has been experimentally realized and studied, as far as we know, only in the case of modulation of gravity by vertical oscillations of a vessel with liquid [4, 5].

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

In our experiments a weak electrolyte filled a wide cylindrical vessel with a transparent bottom. The surface of the electrolyte either remained free or was covered with a layer of liquid dielectric. Above the surface of the electrolyte a semitransparent electrode was installed horizontally. Between the electrolyte and the electrode an alternating potential difference was created with a frequency of $20 \div 100$ Hz and an amplitude of up to 15 kV, with a gap of about 1 cm. The experimental conditions ensured high uniformity of the field in the gap.

The voltage source was a high-voltage transformer powered by a machine generator. The instability was observed by means of a Schlieren system with a vertical optical axis. The system made it possible to measure the wave amplitude starting from 10^{-3}—10^{-2} mm.

Fig. 2

At low strengths of the alternating field, the electrolyte under the electrode rises in the same way as occurs in a constant field ([1]). This rise is due to the nonzero time-average value of the electric force. The surface of the liquid under the electrode remains flat and immobile. At the critical field strength the surface of the electrolyte becomes unstable, and a standing wave arises, close in form to a plane wave. Stroboscopic observations showed that the frequency of the wave coincides with the field frequency, i.e., is equal to half the frequency of modulation of the parameter. This circumstance, as well as the threshold character of the onset of the wave, testify to the parametric mechanism of excitation of the instability. With increasing field strength the wave amplitude increases sharply, and the one-dimensional wave is replaced by a two-dimensional one.

Fig. 3

Figure 1 shows a typical Schlieren photograph of such a wave on the free surface of tap water at a field frequency of 50 Hz. The photograph was obtained with light shuttering at the field frequency. The distance between neighboring bright lines is equal to the wavelength. In the lower left corner of the photograph, remnants of the one-dimensional structure of the wave field are visible. With further increase of the field strength the wave structure becomes more ···

complex (see Fig. 2), then the Schlieren pattern ceases to be stable, and, finally, electrical breakdown occurs between the electrolyte and the electrode.

The investigations showed that the critical field strength at which parametric waves arise increases with the field frequency. In Fig. 3 the abscissa gives the square root of the field frequency, and the ordinate gives the critical field strength. In these coordinates the experimental points fall on a straight line, which is in qualitative agreement with the calculation. The experimental values of the threshold field strength, however, proved to be 30-35% greater than those calculated from formula (2). Estimates showed that such a discrepancy between experiment and calculation can be explained by the strong dependence of the threshold field strength on the properties of the liquid surface, which becomes contaminated during the experiment by products of electrode electrodispersion.

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Note: Figure translations are in progress. See original paper for figures.