INVESTIGATION OF THE GAS-DYNAMIC STABILITY OF A DETONATION WAVE OF ARBITRARY PROFILE

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Fig. 1

Figure 1: Fig. 1

Abstract

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INVESTIGATION OF THE GAS-DYNAMIC STABILITY OF A DETONATION WAVE OF ARBITRARY PROFILE

The instability of plane detonation has been observed theoretically and experimentally in many gaseous and liquid explosives ([1–10]). In all cases it is associated with an Arrhenius dependence of the rate of heat release on temperature. Meanwhile, this is not the only type of chemical-reaction kinetics, especially in solid explosives. In them, for example, kinetics decreasing with time is possible, the origin of which is still not very clear ([9]).

It is of interest to investigate the stability of detonation in the general case, for arbitrary kinetics of heat release. For this purpose a model is proposed in which the smooth variation of the parameters in the detonation front (pressure, density, temperature) is replaced by a stepwise one. In accordance with this, the detonation wave is represented by a model of a stationary complex that includes the initiating shock front and a system of fronts of instantaneously occurring chemical reactions arranged successively behind it, separated by intervals $L_1, L_2, ...$ with characteristic delay times $\tau_1, \tau_2, ...$. The links themselves of the stepwise distribution of parameters within the detonation zone should be understood either as successive stages of one and the same chemical transformation, or as different intermediate reactions with activation energies $E_1, E_2, ...$ (Fig. 1).

Fig. 1

While reflecting the essential features of the real phenomenon, this scheme admits the possibility of a mathematical solution of the stability problem. Increasing the number of intermediate links, within which partial burning-out occurs of the reactant determining the chemical reaction at the given stage (from concentration ψ_j to ψ_{j+1}), naturally increases the accuracy of the calculation. Identifying the axis y with the stationary position of the shock wave, we have in the intervals $L_1, L_2, ...$ a set of flows moving in the positive direction of the x-axis, whose parameters $p_j, \rho_j, v_j, T_j, S_j, a_j, c_{pj}, \varkappa_j$ (pressure, density, velocity, temperature, entropy, speed of sound, heat capacity, isentropic exponent) are denoted by the indices $j = 0$ for the initially explosive substance ($x < 0$), $j = 1, 2, ...$ for the intervals $L_1, L_2, ...$ introduced above in the detonation zone, ending with ignition fronts $x = L_1,\ x = L_1 + L_2, ...$. Perturbed states of the flows occurring downstream from the shock front (for $v_0 > a_0$), are composed of acoustic and entropy-vortex waves, which are represented by solutions of the linearized gas-dynamic equations ([4]) in the form

$$v'_{jx} = A_{j1}\Phi_{j1} + A_{j2}\Phi_{j2} + A_{j3}\psi_j;$$

$$v'_{jy} = \frac{i}{\gamma_{j1}}A_{j1}\Phi_{j1} + \frac{i}{\gamma_{j2}}A_{j2}\Phi_{j2} - \frac{\omega}{kv_j}A_{j3}\psi_j;$$

$$\frac{p'_j}{\rho_j v_j} = \left(\frac{i\omega}{kv_j\gamma_{j1}} - 1\right)A_{j1}\Phi_{j1} + \left(\frac{i\omega}{kv_j\gamma_{j2}} - 1\right)A_{j2}\Phi_{j2}; \qquad (1)$$

$$S'_j = -\frac{c_{pj}}{v_j}A_{j4}\psi_j; \qquad \Phi_{js} = \psi_0 \exp(k\gamma_{js}x); \qquad s = 1, 2.$$

$$\psi_j = \psi_0 \exp\left(\frac{i\omega}{v_j}x\right); \qquad (1 - M_j^2)\gamma_{js} = z_j M_j^2 + (-1)^{s+1}\sqrt{1 - M_j^2 + z_j^2 M_j^2};$$

$$z_j = -i\omega/kv_j; \qquad M_j = v_j/a_j; \qquad j = 1, ..., n+1.$$

In this case the shock front and the fronts of partial combustion introduced by us acquire small displacements

$$\varepsilon_j = \frac{A_{js}}{kv_j}\psi_0.$$

On the acoustic perturbation going into the final detonation products we impose the condition that it be bounded as $x \to +\infty$ ($A_{n+1,1} = 0$). Satisfying, by means of the indicated solutions, the linearized laws of conservation of mass, momentum vector, and energy with respect to the discontinuities,

$$v_{jx} - (1 - \alpha_j)\frac{\partial \varepsilon_j}{\partial t} + \frac{p'_j}{\rho_j v_j}M_j^2 - \frac{v_j}{c_{pj}}S'_j =$$

$$= \alpha_j\left(v'_{j-1,x} + \frac{p'_{j-1}}{\rho_{j-1}v_{j-1}}M_{j-1}^2 - \frac{v_{j-1}}{c_{p,j-1}}S'_{j-1}\right);$$

$$\frac{p_j'}{\rho_j v_j}(1+M_j^2)+2v_{jx}'-v_j\frac{S_j'}{c_{pj}}=\frac{p_{j-1}'}{\rho_{j-1}v_{j-1}}(1+M_{j-1}^2)+2v_{j-1,x}'-\frac{v_{j-1}}{c_{p,j-1}}S_{j-1}'; \quad (2)$$

$$v_{jy}'+v_{j-1}\frac{\partial\varepsilon_j}{\partial y}(\alpha_j-1)=v_{j-1,y}',$$

$$v_{jx}'-\left(1-\frac{1}{\alpha_j}\right)\frac{\partial\varepsilon_j}{\partial t}+\frac{p_j'}{\rho_j v_j}+\frac{v_j}{c_{pj}}\frac{S_j'}{(\chi_j-1)M_j^2}=$$

$$=\frac{1}{\alpha_j}\left[v_{j-1,x}'+\frac{p_{j-1}'}{\rho_{j-1}v_{j-1}}+\frac{v_{j-1}}{c_{p,j-1}}\frac{S_{j-1}'}{(\chi_{j-1}-1)M_{j-1}^2}\right];$$

$$p_0'=v_{0x}'=v_{0y}'=S_0'=0; \qquad \alpha_j=\rho_{j-1}/\rho_j.$$

and the conditions, analogous to (7), imposed by the chemical kinetics on each interval of partial reaction, in the form

$$\int_{T_j}^{T_{j+1}}\left[m_j M_j^2\frac{p_j'}{\rho_j v_j}+N_j\frac{v_j}{c_{pj}}S_j'-v_{jx}'\right]dt'-\varepsilon_j(T_j)+\varepsilon_{j+1}(T_{j+1})=0, \quad (3)$$

where $T_j=t+\sum_{k=0}^{j-1}\tau_k$, $\tau_0=0$, and the integration is carried out along the trajectory

$$x=\sum_{k=1}^{j-1}L_k+v_j(t-\tau_j), \qquad L_k=v_k\tau_k;$$

$$m_j=\chi_j Q_j+(\chi_j-1)N_j;$$

$$Q_j=\partial\ln f/\partial\ln\rho\Big|_{\substack{\rho=p_j\\ T=T_j'}}; \qquad N_j=\partial\ln f/\partial\ln T\Big|_{\substack{\rho=p_j\\ T=T_j'}}; \qquad j=1,\dots,n,$$

we ultimately obtain the characteristic equation

$$D(z)=0, \quad (4)$$

for finding the eigenvalues ω. Since the introduction of each interval requires the fulfillment of 5 conditions (4 conservation laws and the equation determined by the kinetics), the determinant $D(z) = |a_{gr}(z)|$, by which the left-hand side of the characteristic equation is expressed, has order $5n+4$, where n is the number of intervals L_j. It is convenient to represent the numbers of the row and column at whose intersection the element a_{gr} stands in the form $q = 5(j-1)+l$ and $r = 5(j-1)+m$, and to introduce the notation $a_{gr} = b^j_{lm}$, $r, g = 1, 2, \dots, 5n+4$; $l, m = 1, 2, \dots, 5n+4$; $j = 1, 2, \dots, n+1$. Then in $D(z)$, for $j = 1, 2, \dots, n$,

$$b^j_{1s} = 1 - M_j(z_j/\gamma_{js}+1); \qquad b^j_{2s} = 2-(1+M_j^2)(z_j/\gamma_{js}+1); \qquad b^j_{3s} = 1/\gamma_{js};$$

$$b^j_{4s} = z_j/\gamma_{js}; \qquad b^j_{5s} = (m_{jM}j^2/\gamma_{js} + 1/(\gamma_{js}+z_j))[\exp kL_j(\gamma_{js}+z_j)-1],$$

where $s = 1, 2$; $b^j_{44} = 1/(\chi_j - 1)M_j^2$; $b^j_{15} = z_j\alpha_j(\alpha_j-1)$; $b^j_{35} = \alpha_j - 1$;

$$b^j_{45} = -z_j(\alpha_j-1); \quad b^j_{55} = \alpha_j; \quad b^j_{13} = b^j_{43} = 1; \quad b^j_{23} = 2; \quad b^j_{33} = -z_j;$$

$$b^j_{14} = b^j_{24} = -1; \quad b^j_{53} = kL_j; \quad b^j_{54} = -N_{jkL}j; \quad b^j_{5,10} = \exp kL_{jz}j;$$

$$b^j_{5+l,s} = b^j_{l,s}\exp(-kL_j\gamma_{js}); \qquad s = 1, 2, \quad l = 2, 3;$$

$$b^j_{5+l,s} = b^j_{l,s}\exp(-kL_{jz}j), \qquad s = 3, 4, \quad l = 2, 3;$$

$$b^j_{5+l,s} = b^j_{l,s}\alpha_j^{(-1)^{l+1}}\exp(kL_j\gamma_{js}), \qquad s = 1, 2, \quad l = 1, 4.$$

Finally, for $j = n+1$,

$$b^{n+1}_{11} = 1 - M_{n+1}(z_{n+1}/\gamma_{n+1,2}+1); \qquad b^{n+1}_{21} = 2-(1+M^2_{n+1})(z_{n+1}/\gamma_{n+1,2}+1);$$

$$b^{n+1}_{12} = b^{n+1}_{42} = 1; \quad b^{n+1}_{13} = b^{n+1}_{23} = 1; \quad b^{n+1}_{14} = z_{n+1}(\alpha_{n+1}-1)\alpha_{n+1};$$

$$b^{n+1}_{22} = 2; \quad b^{n+1}_{31} = 1/\gamma_{n+1,2}; \quad b^{n+1}_{43} = 1/(\chi_{n+1}-1)M^2_{n+1};$$

$$b_{32}^{n+1} = -z_{n+1}; \quad b_{34}^{n+1} = \alpha_{n+1} - 1; \quad b_{41}^{n+1} = -z_{n+1}/\gamma_{n+1,2}.$$

The remaining elements not covered by these formulas are zero. Analysis of the roots of the obtained characteristic equation makes it possible to judge the stability of the detonation wave for various distributions of the parameters over its width. In the limiting case $L_1 \neq 0,\ L_2 = L_3 = \cdots = L_n = 0$, as was to be expected, the equation coincides exactly with that obtained and investigated earlier in [4] for the two-front model.

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REFERENCES

1. K. I. Shchelkin, ZhETF, **36**, No. 2 (1959).
2. K. I. Shchelkin, DAN, **160**, No. 5 (1965).
3. K. I. Shchelkin, Ya. K. Troshin, *Gas Dynamics of Combustion*, Publishing House of the Academy of Sciences of the USSR, 1963.
4. S. K. Aslanov, DAN, **163**, No. 3 (1965).
5. S. K. Aslanov, DAN, **169**, No. 2 (1966).
6. S. K. Aslanov, *Applied Mechanics*, **3**, No. 3 (1967).
7. R. M. Zaidel, DAN, **136**, No. 5 (1961).
8. R. M. Zaidel, *Applied Mechanics and Technical Physics*, **62**, No. 5 (1962).
9. V. A. Veretennikov, A. I. Dremin et al., *Physics of Combustion and Explosion*, **3**, No. 1 (1967).
10. V. V. Pukhnachev, DAN, **149**, No. 4 (1963).

Note: Figure translations are in progress. See original paper for figures.