SPONTANEOUS EXCITATION OF MAGNETIC FIELDS

PHYSICS

1968

SovietRxiv

Abstract

Full Text

UDC 533.9

PHYSICS

V. N. TSYTOVICH

SPONTANEOUS EXCITATION OF MAGNETIC FIELDS

IN A TURBULENT PLASMA

(Presented by Academician E. K. Zavoisky, 17 VII 1967)

1. It is known ([1]) that in a conducting turbulent fluid magnetic fields can be spontaneously excited, i.e., a turbulent conducting fluid is unstable with respect to the appearance of perturbations carrying magnetic fields. The energy of the magnetic field is drawn from the hydrodynamic turbulent pulsations of the fluid.

The purpose of the present work is to show that magnetic fields can also be spontaneously excited in a turbulent plasma, and to indicate a general method for studying the stability problems of a turbulent plasma.

Let us consider the classical example of a plasma in which intense Langmuir turbulent oscillations are excited. The turbulent pulsations are potential, and therefore magnetic fields are absent. In the present work it is shown that even for an isotropic distribution of Langmuir pulsations a turbulent plasma can become unstable with respect to perturbations whose basic energy is contained in the energy of the magnetic field. The instability of a turbulent plasma can be described by the dielectric permittivity $\varepsilon_{ij}(\omega, \mathbf{k}, W^l_{\mathbf{k}1})$, where $W^l_{\mathbf{k}1}$ is the spectral energy density of Langmuir pulsations, $W^l = \int W^l_{\mathbf{k}1} d\mathbf{k}_1$, on which ε_{ij} depends functionally. While assuming $W^l/nT \ll 1$, we shall at the same time not assume that ε_{ij} is expandable in powers of W^l. The latter is connected with the fact that the instability of interest to us arises in the low-frequency region, much smaller than the effective frequencies $\nu_{eff}(W^l_{\mathbf{k}})$ of collisions of turbulent pulsations with one another and with plasma particles, and therefore this collision integral, which depends on $W^l_{\mathbf{k}}$, cannot be taken into account by perturbation theory. In this sense the ε_{ij} found below is the sum of a perturbation-theory series in $W^l_{\mathbf{k}1}$, corresponding to the first term of an expansion in the parameter $\omega/\nu^{Turb}_{eff} \ll 1$. Pair collisions of particles are neglected, i.e., $\omega/\nu_{coll} \gg 1$. This is possible for $\nu^{Turb}_{eff}(W^l_{\mathbf{k}1}) \gg \nu_{coll}$, which is often satisfied both for turbulent plasma created under laboratory conditions and for plasma under cosmic conditions.

2. Let us split the distribution function and the electromagnetic field into turbulent and regular parts, $f = f^T + f^R$, $\mathbf{E} = \mathbf{E}^T + \mathbf{E}^R$, so that the statisticalensemble averages are $\langle f^T \rangle = 0$, $\langle \mathbf{E}^T \rangle = 0$. From the collisionless kinetic equation it is easy to obtain:

$$\partial f^R/\partial t + \mathbf{v}\,\partial f^R/\partial \mathbf{r} + \mathbf{F}^R\,\partial f^R/\partial \mathbf{v} = -\langle \mathbf{F}^T \partial f^T/\partial \mathbf{v}\rangle \equiv I^T; \tag{1}$$

$$\partial f^T/\partial t + \mathbf{v}\partial f^T/\partial \mathbf{r} + \mathbf{F}^R \partial f^T/\partial \mathbf{v} + \mathbf{F}^T \partial f^R/\partial \mathbf{v} + \mathbf{F}^T \partial f^T/\partial \mathbf{v} - \langle \mathbf{F}^T \partial f^T/\partial \mathbf{v}\rangle = 0, \tag{2}$$

where $\mathbf{F} = \dfrac{e}{m}(\mathbf{E} + [\mathbf{vH}])$; $c = 1$. The right-hand side of (1) describes collisions of particles and turbulent pulsations. A concrete expression for I^T can be obtained by expanding f^T in powers of $\mathbf{E}^T$. The first term corresponds to the quasilinear approximation ([2]) and describes the processes of induced emission and absorption of plasmons by plasma particles; the next term describes induced scattering of turbulent pulsations by plasma particles ([3]).

Consider a state in which the quasilinear collision integral vanishes approximately. This occurs if the phase velocity of the Langmuir pulsations is sufficiently large, so that their interaction with resonant particles is negligibly small. In the subsequent orders in the turbulent field, the collision integral, as the analysis shows, for an isotropic distribution of waves and particles gives only a small change, in the approximation $W^l/nT \ll 1$, of the distribution function for particles of small velocities $v \ll v_{Te}$.

Let the initial turbulent state be regarded, approximately, as homogeneous and stationary, i.e.,

$$\frac{\partial}{\partial t} W^l_{\mathbf{k}} \approx 0 \quad \text{and} \quad I^T \approx 0.$$

Consider a certain perturbation (and the associated field E^R), owing to which both the particle distribution and the pulsation distribution become weakly inhomogeneous and weakly nonstationary. As a result, $I^T \neq 0$. Expanding all quantities in E^R and neglecting terms $W^l/nT \ll 1$, we find from (1)

$$-i(\omega - \mathbf{kv})\,\delta f^R_{\mathbf{k}} + F^R_{\mathbf{k}} \frac{\partial}{\partial \mathbf{v}} f^R_0 = \frac{\partial}{\partial v_i} D_{ij} \frac{\partial}{\partial v_j} f^R_0; \tag{3}$$

$$D_{ij} = i\frac{(4\pi)^2 e^4}{m_e^3} \int \frac{W^l_{\mathbf{k}1}\, d\mathbf{k}_1\, \Lambda^*_{ij}}{\varepsilon^l\{-\omega(\mathbf{k}_1) + \omega,\, -\mathbf{k}_1 + \mathbf{k}\}} \int d\mathbf{v}' \times$$
$$\times \frac{1}{\omega(\mathbf{k}_1) - \mathbf{k}_1\mathbf{v}' - \omega(\mathbf{k}) + \mathbf{kv}' - i\delta} \left\{ \left(\mathbf{k}_1 \frac{\partial}{\partial \mathbf{v}'}\right) \delta f^R_{\mathbf{k}1}(\mathbf{v}') + \right.$$
$$\left. + i\left(F^R_{\mathbf{k}}(\mathbf{v}')\frac{\partial}{\partial \mathbf{v}'}\right) \frac{1}{\omega(\mathbf{k}_1) - \mathbf{k}_1\mathbf{v}' - i\delta} \left(\mathbf{k}_1 \frac{\partial}{\partial \mathbf{v}'}\right) f^R_0(\mathbf{v}') \right\}; \tag{4}$$

$$\Lambda_{ij} = \left\{ \frac{k_{1i}(k_{1j} - k_j)}{k_1^2(\mathbf{k} - \mathbf{k}_1)^2} \frac{1}{\omega(\mathbf{k}_1) - \mathbf{k}_1\mathbf{v} - \omega(\mathbf{k}) + \mathbf{k}_1\mathbf{v} - i\delta} - \right.$$

$$-\frac{k_{1j}(k_{1i}-k_i)}{k_1^2(\mathbf{k}-\mathbf{k}_1)^2}\frac{1}{\omega(\mathbf{k}_1)-\mathbf{k}_1\mathbf{v}+i\delta}\Bigg\},\quad \delta\to+0. \tag{5}$$

Here $\omega(\mathbf{k}_1)$ is the spectrum of the turbulent pulsations; $k=\{\mathbf{k},\omega\}$; $f^R = f_0^R+\delta f^R$; $\delta f_{\mathbf{k}}^R$ is the Fourier component of δf^R; ε^l is the longitudinal linear dielectric permittivity of the plasma. In (3)–(5) only terms containing $1/\varepsilon^l$ (virtual longitudinal waves, see ([3])), which have singularities at small frequencies, have been retained. The solution of the integral equation (3), (4) for $\delta f_{\mathbf{k}}^R$ can be found in the limits $\omega \ll \omega(\mathbf{k}_1)$, $k \ll k_1$. Calculating the current $\mathbf{j}_{\mathbf{k}} = \int e\mathbf{v}\,\delta f_{\mathbf{k}}^R\,d\mathbf{v}$, we find the dielectric tensor

$$\begin{aligned}
\varepsilon_{ij} = \delta_{ij} &+ \frac{4\pi e^2}{\omega m_e}\int\frac{v_i}{\omega-\mathbf{kv}+i\delta}\left[\delta_{jl}\left(1-\frac{\mathbf{kv}}{\omega}\right)+\frac{k_l v_j}{\omega}\right]\frac{\partial f_0^R}{\partial v_l}\,d\mathbf{v}+\\
&+\frac{4\pi e^2}{m_e\omega}\int\frac{v_i}{\omega-\mathbf{kv}+i\delta}\frac{\partial}{\partial v_l}(\omega-\mathbf{kv})\frac{\partial}{\partial v_s}f_0^R(\mathbf{v})\,d\mathbf{v}\times\\
&\times\Bigg\{d_{ls}\int\frac{d\mathbf{v}'}{\omega-\mathbf{kv}'+i\delta}\left[\delta_{mj}\left(1-\frac{\mathbf{kv}'}{\omega}\right)+\frac{k_m v_j'}{\omega}\right]\times\\
&\times\left[\frac{\partial f_0^R(\mathbf{v}')}{\partial v_m'}+d_{nqm}\frac{\partial}{\partial v_n'}(\omega-\mathbf{kv}')\frac{\partial}{\partial v_q'}f_0^R(\mathbf{v}')\right]\times\\
&\times\left[1-d_{rp}\int\frac{d\mathbf{v}''}{\omega-\mathbf{kv}''+i\delta}\frac{\partial}{\partial v_r''}(\omega-\mathbf{kv}'')\frac{\partial f_0^R}{\partial v_p''}\right]^{-1}+\\
&+d_{lsn}\left[\delta_{nj}\left(1-\frac{\mathbf{kv}}{\omega}\right)+\frac{k_n v_j}{\omega}\right]\Bigg\},
\end{aligned} \tag{6}$$

where

$$d_{ij} = -\frac{e^2\pi}{m_e^2 n_0}\int d\mathbf{k}_1\,\frac{k_{1i}k_{1j}}{k_1^2}\frac{(\mathbf{k}\,\partial/\partial\mathbf{k}_1)\,W_{\mathbf{k}1}^l\,[\omega(\mathbf{k}_1)]^{-1}}{\omega-\mathbf{kv}_{gr}+i\delta}\,. \tag{7}$$

$$\begin{aligned}
d_{ijl} = &\frac{12\pi e^2}{m_e^2\omega_{0e}^3}\int d\mathbf{k}_1\,\frac{k_{1i}k_{1j}k_{1l}W_{\mathbf{k}1}^l}{k_1^2\omega(\mathbf{k}_1)\,\partial\varepsilon^l/\partial\omega\big|_{\omega=\omega(\mathbf{k}1)}(\omega-\mathbf{kv}_{gr}+i\delta)}\\
&+\frac{e^2\pi}{m_e^2}\frac{\omega_{0e}}{\omega}\int d\mathbf{k}_1\,\frac{k_{1i}k_{1j}k_{1s}}{k_1^4}\,k_s\left(\delta_{sl}-\frac{k_{1s}k_{1l}}{k_1^2}\right)\\
&\times\frac{1}{\omega-\mathbf{kv}_{gr}+i\delta}\left(\mathbf{k}\frac{\partial}{\partial\mathbf{k}_1}\right)W_{\mathbf{k}1}^l[\omega(\mathbf{k}_1)]^{-1}.
\end{aligned} \tag{8}$$

Here $\mathbf{v}_{gr} = d\omega(\mathbf{k}_1)/d\mathbf{k}_1$ is the group velocity of the turbulent pulsations.

Result (6) is readily generalized to the case of an inhomogeneous and magnetically active plasma. An important conclusion following directly from (6) is that the low-frequency electromagnetic properties of a turbulent plasma differ fundamentally from those of a nonturbulent one.*

3. Let us consider the development of perturbations in a turbulent plasma with an isotropic distribution of particles and turbulent pulsations. Owing to the assumed isotropy, the propagation of perturbations carrying magnetic fields is described by the equation $k^2 = \omega^2 \varepsilon^t_{Turb}$, where ε^t_{Turb} is the transverse dielectric permittivity of the turbulent plasma. From (6)–(8), in the limit $\omega \gg kv_{gr}$ and $\omega \ll kv_{Te}$, we obtain the dispersion equation

$$k^2 = i\sqrt{\frac{\pi}{2}}\frac{\omega_{0e}^2\omega}{kv_{Te}}\left(1 + \frac{k^4}{6\omega^2 n_0 m_e}\int \frac{d\mathbf{k}_1}{k_1^2} W^l_{\mathbf{k}1}\right). \tag{9}$$

Its solution always contains an unstable root

$$\omega = i\sqrt{\frac{1}{2\pi}}\frac{k^3 v_{Te}}{\omega_{0e}^2}\left(\sqrt{1 + 4\sqrt{\frac{\pi}{2}}\frac{v^t_*\omega_{0e}^2}{v_{Te}k^2}} - 1\right), \tag{10}$$

where

$$v^t_* = v_{Te}\int \frac{W^l_{\mathbf{k}1}}{2n_0 T_e} v_\Phi^2 \, d\mathbf{k}_1, \qquad v_\Phi = \frac{\omega_{0e}}{|\mathbf{k}_1|}. \tag{11}$$

In the limit $v^t_* \ll v_{Te}\omega_{0e}^2/k^2$ we have a solution of the type of a periodically growing second sound, $\omega = ikv^t_*$, while for $v^t_* \gg v_{Te}\omega_{0e}^2/k^2$,

$$\omega = i\left(\frac{2}{\pi}\right)^{1/4}\frac{k_1^2\sqrt{v_{Te}v^t_*}}{\omega_{0e}}. \tag{12}$$

Let us note that, in the perturbations considered, $|\varepsilon^t_{Turb}| \gg 1$, i.e. $|H| = |\varepsilon^t E| \gg |E|$.

The analysis shows that for $\omega \ll kv_{gr}$, $\omega \ll kv_{Te}$, growing solutions do not arise. However, the change in the dispersion properties of the plasma may lead to an increase in the thickness of the skin layer when a low-frequency electromagnetic field penetrates into the plasma. Solving (9) for k, we obtain, when the inequality

$$\int \frac{W^l_{\mathbf{k}1}\, d\mathbf{k}_1}{6n_0 T_e} \gg \left(\frac{v_{Te}}{c}\right)^{2/3}\left(\frac{c}{v_\Phi}\right)^2 16\left(\frac{2}{\pi}\right)^{2/3}\left(\frac{\omega}{\omega_{0e}}\right)^{2/3} \tag{13}$$

is satisfied, and $\omega \gg kv_{gr}$, expressions for the skin-layer thickness δ

$$\delta = \frac{1}{\operatorname{Im} k(\omega)} = \frac{c}{\omega_{0e}}\sqrt{\frac{\nu^{Turb}_{eff}}{\omega}}, \qquad \nu^{Turb}_{eff} = \omega_{0e}\left\{\int \frac{v_\Phi^2 W^l_{\mathbf{k}1}\, d\mathbf{k}_1}{6n_0 m_{ec}^2}\right\}^{1/2}. \tag{14}$$

When the inequality inverse to (13) is satisfied,

$$k(\omega) = -i\omega\sqrt{\frac{2}{\pi}\, 6n_0 m_{ec}^2 \Big/ \int \frac{d\mathbf{k}_1}{k_1} v_\Phi^2 W_{\mathbf{k}1}^l}. \tag{15}$$

* In particular, drift-type instabilities associated with plasma inhomogeneity are modified, which is important for questions of confinement of a turbulent plasma.

Along with solution (15), there is a solution which is obtained also in the absence of turbulence. Solution (15) describes a field growing in space, whereas the second is the usual anomalous skin effect.* The possibility of spatial amplification of the field is associated with the consideration of the effects of excitation of magnetic fields. In the limit $\omega \ll kv_{gr}$, when such excitation is impossible, the skin effect, for

$$\int \frac{v_\Phi^4}{v_{Te}^4} \frac{W_{\mathbf{k}1}^l \, d\mathbf{k}_1}{54 n_0 m_e c^2} \gg \left(\frac{\omega_{0e}}{\omega}\right)^{2/3} \left(\frac{v_{Te}}{c}\right)^{4/3} \tag{16}$$

is described by the formula

$$\delta = \frac{1}{\operatorname{Im} k(\omega)} = \frac{c}{\omega_{0e}} \left(\int \frac{v_\Phi^4}{v_{Te}^4} \frac{W_{\mathbf{k}1}^l \, d\mathbf{k}_1}{54 n_0 m_e c^2}\right)^{1/2}. \tag{17}$$

Finally, when the inequality $\omega \gg kv_{Te}$ is satisfied, excitation of magnetic fields is possible only if $\dfrac{v_\Phi}{c} \gg \sqrt{\dfrac{c}{v_{Te}}}$, and, for $\displaystyle\int \frac{W_{\mathbf{k}1}^l \, d\mathbf{k}_1 \, v_\Phi^2}{n_0 m_e c^2} \ll 1$, is described by the formula

$$\omega = ikv_{**}^t, \qquad v_{**}^t = \left\{\int \frac{4\pi v_\Phi^2 W_{\mathbf{k}1}^l}{3 n_0 m_e c^2} \, d\mathbf{k}_1\right\}^{1/2}. \tag{18}$$

Under these conditions the skin effect is also described by (14) with $v_{**}^t(\omega/\omega_{0e})^2$.

4. Effects of the next order in the energy of turbulent pulsations in the integral I^T become comparable with those taken into account, as the analysis shows, when the inequality

$$\omega \lesssim \delta\omega_{\mathbf{k}} = \int \frac{W_{\mathbf{k}1} \, d\mathbf{k}_1}{n_0 T_e} \frac{\varepsilon_i(\omega_{\mathbf{k}} - \omega_{\mathbf{k}1}, \, \mathbf{k} - \mathbf{k}_1)}{\varepsilon(\omega_{\mathbf{k}} - \omega_{\mathbf{k}1}, \, \mathbf{k} - \mathbf{k}_1)}$$

is satisfied, i.e., when the frequency becomes comparable with the characteristic correlation frequency of the turbulent pulsations. The condition $\omega \gg \delta\omega_{\mathbf{k}}$ does not impose severe restrictions on the considered effects of spontaneous excitation of magnetic fields; however, it proves to be severe for instabilities of turbulent plasma considered in (4, 5). It is precisely for this reason that one should suppose that under conditions of turbulent plasma heating (6), manifestations of the above-considered instabilities should be expected.

The author expresses deep gratitude to E. K. Zavoisky for his interest in the present work.

P. N. Lebedev Physical Institute
Academy of Sciences of the USSR

Received
7 V 1967

CITED LITERATURE

1. L. D. Landau, E. M. Lifshitz, *Electrodynamics of Continuous Media*, Moscow, 1957.

2. A. A. Vedenov, E. L. Velikhov, R. Z. Sagdeev, *Nuclear Fusion*, Suppl. 2, 465 (1962).

3. V. N. Tsytovich, *ZhETF*, 90, 435 (1966).

4. A. G. Litvak, Abstract of dissertation, Phys. Inst. im. P. N. Lebedev, Academy of Sciences of the USSR, 1964.

5. A. A. Vedenov, L. I. Rudakov, *DAN*, 159, 767 (1964).

6. M. V. Babykin, P. P. Gavrin, E. K. Zavoisky, L. I. Rudakov et al., *ZhETF*, 46, 514 (1964).

* Strictly speaking, in considering the skin effect one should take into account the boundary conditions both for the particles and for the plasmons.

Note: Figure translations are in progress. See original paper for figures.