ON THE DETERMINATION OF THE LUMINOSITY FUNCTION AND STELLAR DENSITY IN THE SOLAR NEIGHBORHOOD

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Abstract

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ON THE DETERMINATION OF THE LUMINOSITY FUNCTION AND STELLAR DENSITY IN THE SOLAR NEIGHBORHOOD

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The construction of the luminosity function is usually reduced to the choice of a certain sphere with its center at the Sun and to the calculation of the distribution of the number of stars by luminosities within the volume of this sphere. In doing so, a difficulty arises connected with the fact that, for a small radius of the sphere, the sample is not representative, while with an increase in its radius the mean density is artificially lowered and, consequently, so is the value of the function for stars of low luminosity. Therefore the values of the luminosity function at large M are always underestimated.

As for the stellar density in the neighborhood of the Sun, the following procedure is used for its determination. The apparent stellar density $D(r)$ is counted in a series of spherical shells with center at the Sun; then the dependence of D on the distance r is approximated by a linear function, and this function is extrapolated to r equal to zero. Such a method is fundamentally incorrect, since there are no grounds for expecting a linear dependence between the apparent stellar density and distance. Indeed, the decrease in apparent stellar density in passing from some distance r to $r+dr$ occurs because stars of some luminosity M, observable at the given limiting apparent magnitude m at distance r, cease to be observable at distance $r+dr$. Therefore, for the magnitude of the change in apparent stellar density we shall have

$$dD(r) = D_0 d\varphi(M), \tag{1}$$

where r, m, and M are connected by the usual relation

$$m = M + 5\lg r - 5, \tag{2}$$

D_0 is the true stellar density, which is assumed constant, and $\varphi(M)$ is the luminosity function. Hence

$$\frac{dD(r)}{dr} = -5D_0 \lg e \frac{1}{r} \frac{d\varphi(M)}{dM}. \tag{3}$$

Consequently, $dD(r)/dr$ can be constant only in one special case, when the luminosity function decreases exponentially with increasing M.

Below a method is proposed for constructing the luminosity function and calculating the stellar density which, as it seems to us, is free from the shortcomings of the methods described.

We shall proceed from the assumption that the spatial distribution of stars in the neighborhood of the Sun follows Poisson' s law. Then, for the mean value of the cube of the distance of the nearest star to the Sun of a given luminosity M, we shall have

$$\frac{4}{3}\pi r_1^3 = 1/D_0(M), \tag{4}$$

where $D_o(M)$ is the density due to stars of luminosity M. Thus, in principle, the partial stellar densities due to stars of different luminosities can be determined, using equality (4), from only the distances of the nearest stars of the given luminosity. However, in order to reduce the influence of random fluctuations, it is reasonable to include other stars of the given luminosity in determining the partial densities.

It is easy to show that for the mean value of the cube of the distance of the k-th star (in order of increasing distance) the equality

$$^4/_3\pi\overline{r_k^3} = \frac{k}{D_o(M)}. \tag{5}$$

will hold. Therefore, if the distances of the n nearest stars of a given luminosity M are known, then $D_o(M)$ can be computed from the equality

$$D_o(M) = \frac{1}{^4/_3\pi n} \sum_{k=1}^{n} \frac{k}{r_k^3}. \tag{6}$$

It is meaningful, however, to take into account the weights of the individual determinations $D_{ok}(M)$ for different k. It is easy to show that the variance of r_k^3 is expressed as

$$\sigma^2(r_k^3) = r_k^6/k. \tag{7}$$

Therefore, using the approximate representation of the variance of a function by means of the variance of an independent random variable, which is quite sufficient for our purpose (see, for example, ([1])), we obtain

Fig. 1

Figure 1: Fig. 1

$$\sigma^2[D_{ok}(M)] = \frac{k}{\left({}^4/{}_3\pi r_k^3\right)^2}. \tag{8}$$

Fig. 1

Finally, assigning to each determination $D_{ok}(M)$ a weight inversely proportional to $\sigma[D_{ok}(M)]$, we obtain

$$D_o(M) = \sum_{k=1}^{n} \sqrt{k} \Bigg/ \frac{4}{3}\pi \sum_{k=1}^{n} \frac{r_k^3}{\sqrt{k}}. \tag{9}$$

Next, the total stellar density is determined from the equality

$$D_o = \int_{-\infty}^{\infty} D_o(M)\, dM, \tag{10}$$

and the value of the luminosity function from

$$\varphi(M) = D_o(M)/D_o. \tag{11}$$

The advantage of the proposed method for determining the luminosity function is that it does not restrict the possibility of using, for determining the partial stellar densities corresponding to different luminosities, data on stars located at different distances. As the luminosity increases, the radius of the sphere that can be used to determine the partial density due to stars of this luminosity increases correspondingly.

As for the total stellar density, it can also be calculated directly from formula (9), if all the nearest stars are taken into account, irrespective of their luminosity. In doing so, however, it is necessary to take into account only stars from a sphere of sufficiently small radius, so that all the nearest stars are included. For a reliable choice of such a sphere one may, for example, first compute a series of mean values of the form (6) (for $n = 1, 2, ...$) and then restrict oneself to those distance values for which the deviations of the individual values D_{ok} from the means do not exceed the quantities $\sigma(D_{ok})$, determined by formula (8). This will mean that the deviations from the mean are due to natural fluctuations of the density, and not to a decrease in the completeness of the data with increasing distance.

In conclusion, for illustration, we give the result of determining the total stellar density, obtained from the material on the nearest stars from the catalogue of W. Gliese ([2]). In Fig. 1, the points mark the densities computed by formula

(5), corresponding to the nearest 50 stars. The solid line represents the course of the mean values as a function of the number of stars, and the dashed lines bound the region between $\overline{D_{ok}} - \sigma(D_{ok})$ and $\overline{D_{ok}} + \sigma(D_{ok})$. The peak in the left part of the figure is due to the triple system α Centauri. It is clearly seen that in the interval $18 \leq k \leq 35$ the mean density value, computed for different k, remains constant, while for $k > 35$ the incompleteness of the data becomes apparent. Therefore, more distant stars cannot be used in computing the density. For the nearest 35 stars, the mean density value is found to be 0.118 ± 0.038 (pc^{-3}). The weighted mean, computed by formula (9), is 0.111 ± 0.003 (pc^{-3}). Analogous results obtained for dwarfs of spectral classes K—M are, respectively, 0.091 ± 0.018 and 0.087 ± 0.003 (pc^{-3}).

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Note: Figure translations are in progress. See original paper for figures.