# A MODEL FOR CALCULATING THE DEPOSITION OF A HEAVY INHOMOGENEOUS IMPURITY FROM A VOLUME SOURCE IN A WIND VARYING WITH HEIGHT

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## Abstract

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A. Ya. PRESSMAN

# A MODEL FOR CALCULATING THE DEPOSITION OF A HEAVY INHOMOGENEOUS IMPURITY FROM A VOLUME SOURCE IN A WIND VARYING WITH HEIGHT

*(Presented by Academician E. K. Fedorov, 9 X 1967)*

Solving the problem of the spread in the atmosphere of an impurity whose particles possess an established settling velocity, in a wind field with velocity variable with height, presents substantial computational difficulties even in the case of an instantaneous point source. In the present note a computational model is set forth, based on physical simplifications that make it possible to bypass a number of difficulties and to obtain a solution of the problem of the deposition on the underlying surface of a heavy polydisperse impurity from a volume source in a wind varying arbitrarily with height. Naturally, the application of such a model is limited by the assumption that the wind velocity is invariant in time.

In papers ([1], [2]) it was shown that, in the scattering in the atmosphere of a heavy impurity with a sufficiently broad set of particle settling velocities $w$, there exist definite spatial and temporal limits within which the scattering of the impurity substance due to the vertical turbulence of the atmosphere may be neglected in comparison with the scattering of the impurity in the vertical due to the difference in the settling velocities of the various fractions. These limits prove to be sufficiently broad to describe the behavior of that part of the impurity which creates the greatest concentrations on the underlying surface. Under this assumption, the motion of particles in the vertical is obviously determined entirely by the dependence

$$z = h - wt,$$

where $h$ is the height from which the particle began to fall, $t$ is time, and $z$ is the vertical coordinate describing the instantaneous position of the particle. The time for a particle to reach the underlying surface ($z = 0$) is equal to $h/w$.

It is important to note that, within the above-mentioned limits, the behavior of an individual weight fraction of the impurity, all particles of which fall with one and the same velocity $w$, can formally be described by the equation of the semiempirical theory of turbulent diffusion (see ([3])), in which the coefficient of vertical turbulent diffusion $K_z$, together with the second derivative with respect to the vertical coordinate, is absent. Such a radical simplification makes it possible to obtain a solution of the equation

$$dc/dt + u_x(z)dc/dx + u_y(z)dc/dy - wdc/dz = K(d^2c/dx^2 + d^2c/dy^2) \quad (1)$$

(where $c$ is the "volume" concentration of particles of the given weight fraction, $K$ is the coefficient of horizontal turbulent diffusion) for the components of the horizontal wind $u_x(z)$ and $u_y(z)$, specified as arbitrary functions of the height $z$ in the interval $0 \le z \le h$. As shown in ([4]), the scattering coefficient of a heavy impurity may be regarded as a quantity inversely proportional to the settling velocity $w$, which in our case means proportionality to the duration of the particles' residence in the atmosphere $t = h/w$.

As shown in ([5]), in the case of an instantaneous point source of unit intensity

$$c\big|_{t=0} = \delta(x)\delta(y)\delta(t-h) \quad (2)$$

the expression for the surface concentration of an individual weight fraction that has fallen onto the underlying surface $z = 0$ has the form

$$p_1(x, y; h, w) =$$

$$= \int_0^\infty wc\big|_{z=0}(x, y, t; h, w)\, dt = \frac{1}{2\pi\sigma^2} \exp\left\{ -\frac{[x - U_{xh}/w]^2 + [y - U_{yh}/w]^2}{2\sigma^2} \right\}, \quad (3)$$

where the variance $\sigma^2$ is a function of $t = h/w$ and is related to the coefficient of horizontal turbulent diffusion $K$ by the relation

$$\sigma^2(t) = 2\int_0^t K(\tau)\, d\tau.$$

In expression (3), $U_x(h)$ and $U_y(h)$ are the components of the velocity of the mean wind, which represents the resultant vector of the horizontal transport of particles as they fall through the layer $0 \le z \le h$. As shown in ([5]),

$$U_x(h) = \frac{1}{h}\int\limits_0^h u_x(z)\,dz, \qquad U_y(h) = \frac{1}{h}\int\limits_0^h u_y(z)\,dz. \tag{5}$$

As in ([6]), we shall assume that

$$\sigma^2(h) = \alpha^2 V^2(h)(h/w)^2, \tag{6}$$

where

$$V^2(h) = U_x^2(h) + U_y^2(h). \tag{7}$$

Let us note that, as follows from formula (3), the distribution on the plane $z = 0$ of the surface concentration $p_1(x, y; h, w)$ possesses circular symmetry with respect to the point of its maximum

$$x_0 = U_x(h)h/w, \qquad y_0 = U_y(h)h/w. \tag{8}$$

From formulas (3) and (8) it follows that all weight fractions that began falling from the same height $h$, independently of the falling velocity $w$, will form on the plane $z = 0$ distributions $p_1(x, y; h, w)$, whose maxima are located on the ray

$$y = xU_y(h)/U_x(h). \tag{9}$$

If the inhomogeneity of the impurity particles in the source is characterized by the distribution density $N(h, w)$ (which, generally speaking, may be different for each height $h$), then the surface concentration of a polydisperse impurity that has fallen from a point source located at height $h$ will have the form of an integral over the entire range of velocities $w$

$$p_2(x, y; h) = \int\limits_0^\infty N(h, w)p_1(x, y; h, w)\,dw. \tag{10}$$

Obviously, the function $p_2(x, y; h)$ will be symmetric with respect to the ray (9), which is the locus of its maxima. In ([6]) the expression $p_2(x, y; h)$ was analyzed when $N(h, w)$ is specified in the form of the two-parameter function

$$N(h, w) = \frac{a^{n+1}}{\Gamma(n+1)} w^n e^{-aw}, \tag{11}$$

where $a(h) > 0$, $n(h) > -1$, which is convenient for describing a sufficiently broad class of real distributions. In this case it is possible to obtain simple

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

analytical expressions that make it easy to calculate the distribution of surface concentration on the plane $z = 0$.

Passing from an instantaneous point source to an instantaneous plane source located at the same height $h$, one must obviously integrate $p_2(x, y; h)$ together with the initial distribution of the impurity in the source $L(\xi, \eta)$, where $\xi$ and $\eta$ are the horizontal coordinates in the plane $z = h$.

Fig. 1

The simplest expressions are obtained in the case when the impurity is distributed in the source at the instant $t = 0$ according to the Gaussian law

$$L(\xi, \eta) = \frac{1}{2\pi\sigma_0^2} \exp\left[-\frac{\xi^2 + \eta^2}{2\sigma_0^2}\right]. \tag{12}$$

Fig. 2

Here the initial variance $\sigma_0^2$, generally speaking, is a function of the source height $h$. Integrating $p_2(x, y; h)$ together with $L(\xi, \eta)$ over the plane $z = h$, we obtain an expression for the surface concentration on the plane $z = 0$

$$p_3(x, y; h) = \iint_{-\infty}^{+\infty} p_2(x - \xi, y - \eta; h) L(\xi, \eta)\, d\xi\, d\eta =$$

$$= \int_0^{\infty} \frac{N(h, w)}{2\pi[\sigma_0^2(h) + \sigma^2(h)]} \exp\left\{-\frac{[x - U_x(h)h/w]^2 + [y - U_y(h)h/w]^2}{2[\sigma_0^2(h) + \sigma^2(h)]}\right\} dw. \tag{13}$$

It is now not difficult to pass to the distribution of surface concentration when the impurity falls out from an instantaneous volume source, by integrating (13) with the density $M(h)$, which characterizes the distribution of the impurity over the source height in the layer $0 \le h \le H$:

$$p_4(x, y) =$$

$$= \int_0^H \left[ \int_0^\infty \frac{M(h)N(h,w)}{2\pi[\sigma_0^2(h)+\sigma^2(h)]} \exp\left\{ -\frac{[x-U_x(h)h/w]^2+[y-U_y(h)h/w]^2}{2[\sigma_0^2(h)+\sigma^2(h)]} \right\} dw \right] dh. \tag{14}$$

As an example of calculation by formula (14), we present qualitative pictures (Figs. 1 and 2) of the distribution on the plane $z = 0$ of the surface concentration of an impurity that has fallen out from a volume source with a uniform distribution of the substance at the initial instant along the vertical, for two hypothetical cases: a wind that turns strongly with height, and a wind whose velocity does not change in direction within the layer $0 \le h \le H$ occupied by the source. Since $\bar{u}_i$ (the wind velocity) is usually specified discretely at $m$ levels $(i = 1, 2, \dots, m)$, in the calculation scheme the distribution $M(h)$ was specified discretely at the same levels. In the calculations it was assumed that $m = 8$. In the first case (Fig. 1), the rays $h_k$ (see formula (9)) correspond to the directions of the mean winds, whose velocities were averaged over $k$ equal layers, beginning from the ground surface,

$$\bar{V}_k = \frac{1}{k}\sum_{i=1}^{k} \bar{u}_i \qquad (k = 1, 2, \dots, 8). \tag{15}$$

Figure 2 shows the distribution pattern of isolines of the surface concentration for a wind, constant in direction along $ox$ throughout the entire layer occupied by the source. In contrast to the point source, when the maximum of the surface concentration is appreciably displaced in the direction of the mean wind, in the present case the maximum is located practically at the origin of the coordinates.

In both examples considered above, for simplicity it was assumed that in the function $N(h, w)$ the parameter $n = 0$, and that the parameters $a$ and $\sigma_0$ are the same for all levels.

Institute
of Applied Geophysics

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*Note: Figure translations are in progress. See original paper for figures.*