INSTABILITY CRITERIA FOR DETONATION WAVES

MECHANICS

1968

SovietRxiv

Abstract

Full Text

UDC 534.222.2

MECHANICS

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INSTABILITY CRITERIA FOR DETONATION WAVES

In [1] a method was proposed for calculating the stability of a detonation wave on the basis of its multistage model. Investigation of the characteristic equation $D(z) = 0$, obtained in [1], makes it possible to find criteria for the instability of detonation waves. In the case $L_1 \approx L_2 \approx \cdots \approx L_n \neq 0$, for perturbations of small relative wavelengths ($kL_j \gg 1$), with accuracy up to $e^{-\delta}$, where $\delta = \min\{kL_j\gamma_{j1}\}$, the determinant

$$D(z) = \prod_{l=0}^{n} D_l(z).$$

The characteristic equation, consequently, splits into a series of equations

$$D_0(z) = 0, \qquad D_j(z) = 0, \qquad j = 1, 2, \dots, n,$$

as was indicated earlier in [2], where

$$D_0(z) = (\alpha_1 - 1)\begin{vmatrix} 1 - M_1^2(z_1/\gamma_{12} + 1) & 1 & -1 & z_1\alpha_1 \\ 2 - (1 + M_1^2)(z_1/\gamma_{12} + 1) & 2 & -1 & 0 \\ 1/\gamma_{12} & -z_1 & 0 & 1 \\ -z_1/\gamma_{12} & 1 & 1/(\alpha_1 - 1)M_1^2 & -z_1 \end{vmatrix}, \quad (1)$$

$$D_j(z) = \begin{vmatrix} a_{11}^j & a_{12}^j & 1 & -1 & z_{j+1}\alpha_{j+1} \\ a_{21}^j & a_{22}^j & 2 & -1 & 0 \\ \dfrac{1}{\gamma_{j1}} & \dfrac{1}{\gamma_{j+1,2}} & -z_{j+1} & 0 & 1 \\ -\dfrac{z_j}{\alpha_j\gamma_{j1}} & -\dfrac{z_{j+1}}{\gamma_{j+1,2}} & 1 & \dfrac{1}{(\alpha_{j+1} - 1)M_{j+1}^2} & -z_{j+1} \\ a_{51}^j & 0 & 0 & 0 & \dfrac{1}{\alpha_{j+1} - 1} \end{vmatrix} \quad (2)$$

$$a_{11}^{j} = \left[1 - M_j^2 \left(\frac{z_j}{\gamma_{j1}} + 1\right)\right] \alpha_j; \qquad a_{12}^{j} = 1 - M_{j+1}^2 \left(\frac{z_{j+1}}{\gamma_{j+1,2}} + 1\right);$$

$$a_{21}^{j} = 2 - (1 + M_j^2) \left(\frac{z_j^2}{\gamma_{j1}} + 1\right); \qquad a_{22}^{j} = 2 - (1 + M_{j+1}^2) \left(\frac{z_{j+1}}{\gamma_{j+1,2}} + 1\right);$$

$$a_{51}^{j} = \frac{m_{jM} j^2}{\gamma_{j1}} + \frac{1}{(\gamma_{j1} + z_j)}; \qquad j = 1, 2, \ldots, n$$

(n is the number of intervals of the stepwise scheme).

The characteristic equation $D_0(z) = 0$ for the shock front, as shown in [3], has no roots with positive real part. Analysis of the remaining equations makes it possible to obtain a sufficient criterion for instability of a detonation wave to perturbations of small relative wavelength in the form of restrictions on the chemical kinetics; namely, it is sufficient that at least one of the inequalities be satisfied.

$$\eta_{1j} < m_j < \eta_{2j}, \tag{3}$$

where

$$\eta_{1j} = \frac{1 + \dfrac{j}{(\alpha_{j+1} - 1)M_j^2}}{\left\{1 - \dfrac{1 + M_{jM_{j+1}}}{(1 + M_j)\left[1 + M_{j+1} + \dfrac{\alpha_{j+1} - 1}{\alpha_{j+1}}(\varkappa_{j+1} - 1)M_{j+1}^2\right]}\right\}}; \tag{4}$$

$$\eta_{2j} = \frac{2 - \alpha_{j+1}}{(\alpha_{j+1} - 1)M_j^2} + \frac{\sqrt{1 - M_j^2}}{(\alpha_{j+1} - 1)M_j^2 \sqrt{1 - M_{j+1}^2}} \times$$

$$\times \left\{1 + (\alpha_{j+1} - 1)\left[2 + (\varkappa_{j+1} - 1)M_{j+1}^2\right]\right\},$$

i.e., the detonation wave will be unstable if at least one of the values m_j lies in the band bounded by the broken curves η_{1j}, η_{2j} (Fig. 1).

Fig. 1

Fig. 1

Fig. 2
Fig. 2

For Chapman–Jouguet detonation, as $x = L$ is approached, the upper boundary of the instability region $\eta_{2j} \to +\infty$. Carrying out the limiting transition as $k \to 0$ ($z \to \infty$, while $\xi = kz$ remains finite), we obtain the characteristic equation for one-dimensional perturbations $A(\xi) = 0$. In unexpanded form $A(\xi)$ is represented by a determinant of order $(4n+3)$. Calculation of $A(0)$ gives the following:

$$A(0) = (-1)^{\beta+1}\left(1 - \frac{\rho_0}{\rho_{n+1}}\right)^2 \frac{1+M_{n+1}}{M_{n+1}} \prod_{l=0}^{n} A_l,$$

where

$$A_0 = \frac{\rho_0}{\rho_{n+1}}(1+M_{n+1}) \Big/ \left(1 - \frac{\rho_0}{\rho_{n+1}}\right) M_{n+1}^2 - 1;$$

$$A_l = \frac{1-M_l^2}{M_l^2}\,\frac{2}{(\varkappa_l - 1)M_l}; \qquad l = 1, \dots, n; \qquad \beta = \frac{n(n+1)}{2}.$$

The sign of $A(\infty)$ is determined by the sign of the coefficient B at the highest exponent

$$\exp\left\{\sum_{j=1}^{n} \frac{\xi_{jL} j}{1 - M_j}\right\},$$

$$B = (-1)^{\beta+1} \prod_{k=0}^{n} B_k, \tag{5}$$

where

$$B_0 = (1-\alpha_1)^2\left(1 + \frac{1}{M_1}\right)\left[\frac{\alpha_1(1+M_1)}{(\chi_1 - 1)(1-\alpha_1)M_1^2} - 1\right];$$

$$B_k = \alpha_{k+1}\begin{vmatrix} M_k(\alpha_{k+1}-1) & \alpha_{k+1}-1 & \alpha_{k+1}/(\chi_{k+1}-1)M_{k+1} & 0 \\ -(1-M_k) & 1+M_{k+1} & -1 & 0 \\ -1 & \alpha_{k+1} & \alpha_{k+1}/(\chi_{k+1}-1)M_{k+1} & -(\alpha_{k+1}-1) \\ M_k[m_{kM}k+1] & 0 & 0 & 1 \end{vmatrix}.$$

Using the gas-dynamic relations of detonation theory, it is easy to show that A_0, A_k, B_0, B_k are positive. Comparing the expressions obtained for $A(0)$ and

Fig. 3

Figure 1: Fig. 3

$A(+\infty)$, we find that their signs are opposite (and this is a sufficient condition for the existence of at least one positive eigenvalue) only when the sign of the product

$$\prod_{k=1}^{n} B_k$$

is negative. From this we obtain a sufficient condition for one-dimensional instability of a detonation wave in the form of the system of inequalities

$$m_i < \eta_{i1}, \qquad m_j > \eta_{j1}, \qquad i \neq j, \tag{6}$$

where η_{j1} is determined by formula (4), and the number of the latter inequalities is odd, i.e., the broken line m_j crosses the broken line η_{j1} from below upward at least once. The range of values of m_j leading to instability in the case of one-dimensional perturbations is not bounded above, unlike the case of short-wave perturbations.

Fig. 3

Criteria (3) and (6), in contrast to the corresponding criteria for the single-stage model ([3]), contain parameters of weaker discontinuities, as a result of which the multistage model proves to be more stable. In the case of different orders of magnitude of L_j (for example, Fig. 2 represents a distribution characteristic of detonation in a gas), when $L_i \ll L_j$, $i \neq j$, $i \neq 1$, the criteria are the same as if, instead of the i-th and j-th fronts of chemical reactions, one resultant front were considered (for the distribution in Fig. 2 the instability criteria coincide with those ([2]) for the single-stage model). When $L_1 \ll L_j$, $j = 2, 3, \dots, n$ (Fig. 3), the parameters of the narrow zone 1 will not enter the criterion (in ([3,6]), $j = 2, 3, \dots$). Thus, a detonation wave with a steep drop in pressure (density) behind the shock front (for example, such a detonation wave in solid explosives may be considered) proves to be much more stable than follows from the single-stage approach ([2]).

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Received
19 IV 1968

CITED LITERATURE

[1] S. K. Aslanov, V. N. Budzirovskii, K. I. Shchelkin, DAN, 182, No. 1 (1968). [2] S. K. Aslanov, Prikl. mekh., 2, issue 7 (1966). [3] S. K. Aslanov, DAN, 169, No. 2 (1966).

Note: Figure translations are in progress. See original paper for figures.