ON THE DEVELOPMENT OF THE WESTERN INTENSIFICATION OF OCEAN CURRENTS. A NUMERICAL EXPERIMENT

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Abstract

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ON THE DEVELOPMENT OF THE WESTERN INTENSIFICATION OF OCEAN CURRENTS. A NUMERICAL EXPERIMENT

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An interesting result obtained in the theory of ocean currents is the explanation of the intensification of established currents near the western coast of the ocean (1). In the present work an attempt is made to obtain a qualitative picture of the emergence and development of this phenomenon.

Let a wind begin to blow over an ocean initially at rest, and let it then remain unchanged in time. We consider, in a linear formulation, the problem of the emergence of ocean currents and of their adjustment to the prescribed wind field. Neglecting the horizontal exchange of momentum and regarding the fluid as homogeneous, i.e., remaining within the framework of the classical theory of marine currents, we write the equations of motion and continuity in the form (2)

$$\begin{aligned} \partial u/\partial t - \Omega v &= g\,\partial\zeta/\partial x + A\,\partial^2 u/\partial z^2; \\ \partial v/\partial t + \Omega u &= g\,\partial\zeta/\partial y + A\,\partial^2 v/\partial z^2; \\ \partial\zeta/\partial t &= \partial S_x/\partial x + \partial S_y/\partial y. \end{aligned} \tag{1}$$

The boundary and initial conditions are as follows:

$$\text{at } z = 0 \quad A\,\partial u/\partial z = -T_x/\rho, \quad A\,\partial v/\partial z = -T_y/\rho; \tag{2}$$

$$\text{at } z = H \quad u = v = 0; \tag{3}$$

$$\text{on the coastal contour} \quad S_n = 0; \tag{4}$$

$$\text{at } t = 0 \quad u = v = \zeta = 0. \tag{5}$$

Figure 1. Curve of the distribution of wind stress with latitude and patterns of integral circulation at different moments in time. a –$5 \cdot 10^3$ cm^2/sec; b – $2.5 \cdot 10^4$ cm^2/sec; c –$5 \cdot 10^4$ cm^2/sec.

Figure 1: Figure 1. Curve of the distribution of wind stress with latitude and patterns of integral circulation at different moments in time. a –$5 \cdot 10^3$ cm^2/sec; b –$2.5 \cdot 10^4$ cm^2/sec; c –$5 \cdot 10^4$ cm^2/sec.

Here the following notation is used: u, v are the horizontal components of the current velocity along the axes X, Y; ζ is the dynamic level of the free surface; g is the acceleration of gravity; $\Omega = \Omega(y) = 2\omega \sin\varphi$ is the Coriolis parameter, where ω is the angular velocity of the Earth' s rotation, φ is latitude, and it is assumed that $\varphi = \dfrac{\pi}{180}\dfrac{y}{\Delta}$ ($\Delta = 14$ km is the length of one degree of latitude); $S_x = \int_0^H u\,dz$, $S_y = \int_0^H v\,dz$, the components of the total transport; A is the kinematic coefficient of turbulent viscosity; $H =$ const is the depth of the layer encompassed by the motion; $\rho = 1$ g $\cdot$ cm^{-3} is the density of seawater; T_x, T_y are the components of the tangential wind stress; n is the direction of the normal to the contour. The X-axis is directed eastward along the equator, the Y-axis northward, and the Z-axis vertically downward. The origin of coordinates is located on the undisturbed sea surface. t is time.

Following Stommel (1), we shall study the motion generated by a zonal wind $T_x = T_x(y)$, $T_y = 0$ in a closed rectangular region located outside the equator,

$$0 \le x \le L, \qquad y_0 \le y \le y_1, \qquad 0 \le z \le H. \tag{6}$$

We solve the problem numerically by means of the method set forth in (2). We use the difference scheme

$$(u^{n+1} - u^n)/\tau - \Omega v^{n+1} = g\,\partial\zeta^{n+1}/\partial x + A\,\partial^2 u^{n+1}/\partial z^2; \tag{7}$$

$$(v^{n+1} - v^n)/\tau + \Omega u^{n+1} = g\,\partial\zeta^{n+1}/\partial y + A\,\partial^2 v^{n+1}/\partial z^2; \tag{8}$$

Fig. 1. Curve of the distribution of wind stress with latitude and patterns of integral circulation at different moments in time.
a –$5 \cdot 10^3$ cm^2/sec; b –$2.5 \cdot 10^4$ cm^2/sec; c –$5 \cdot 10^4$ cm^2/sec.

$$(\xi^{n+1} - \xi^n)/\tau = \partial S_x^n/\partial x + \partial S_y^n/\partial y, \tag{9}$$

where $\tau = t_{n+1} - t_n$ is the time step.

The order of operations is as follows. Suppose that at time n the values u^n, v^n, ξ^n are known. By numerical integration, for example by Simpson' s formula, we compute the components of the total flow S_x^n, S_y^n. Then from equation (10), using an explicit formula, we find the level ξ^{n+1}, as well as its derivatives $\partial\xi^{n+1}/\partial x$ and $\partial\xi^{n+1}/\partial y$. After this, from equations (8), (9), by matrix elimination (see, for example, (3)) we find u^{n+1}, v^{n+1}, etc., on each vertical.

Let us note that at any time one can compute the vertical component of the current velocity by formula (2)

$$w = \int_z^H \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dz. \tag{10}$$

Along the X-axis the region was divided into 16 parts, along the Y-axis into 8, and in depth into 8 parts. The grid in all directions could be arbitrary; however, for simplicity the main calculations were carried out on a uniform grid. The boundary was excluded from the calculation; specifically, the current velocity and the level were computed only in the inner region, at a distance of one step from the boundary. This made it possible to compute the derivatives $\partial S_x/\partial x$ and $\partial S_y/\partial y$ everywhere only by central differences. However, the derivatives $\partial\xi/\partial x$ and $\partial\xi/\partial y$ at the boundary of the inner region had to be computed using one-sided differences. Taking the boundaries into account, as the calculations showed, led to instability, which did not disappear when the time step was reduced. In order to take the boundary condition (2) into account more accurately, a fictitious horizon ($z_{-1} = -\Delta z$) was introduced near the surface, as is customary, and it was assumed that the equation is valid at the ocean surface ($z_0 = 0$).

The program was constructed in such a way that only the components of the total flow S_x and S_y were printed out. This led to a considerable saving of machine time and, at the same time, in our opinion, provided sufficient information about the development of the process. The calculations were carried out on the BESM-4 electronic computer; one time step without printing took 6 sec of machine time. The following parameter values were chosen: $H = 200$ m, i.e. $\Delta z = 25$ m, $L = 5000$ km, i.e. $\Delta x = 315.5$ km, $y_0 = 3330$ km ($\varphi_0 = 30°$), $\Delta y = 555$ km (5°), $A = 100$ cm^2/sec, $g = 980$ cm/sec^2.

The wind stress T_x was taken to vary linearly with latitude,

$$T_x = -T_0(y_0 - 2y + y_1)/(y_1 - y_0),$$

and in the lower half it was negative (east wind), while in the upper half it was positive (west wind), becoming zero in the middle. The wind amplitude T_0 was taken equal to 1 dyne/cm^2. It is easy to see that, owing to the linearity of the problem, the components of the current velocity and the level are directly proportional to the wind amplitude.

The time step was chosen experimentally and was taken equal to 50 min.

Figure 2

Figure 2: Figure 2

The results of the calculations are shown in Figs. 1 and 2. Fig. 2 shows the behavior of the meridional component of the total flow S_y as a function of time at two points located on the middle parallel at the western (curve 1) and eastern (curve 2) boundaries of the inner region. As can be seen from the nature of the behavior of S_y as a function of time, the difference scheme may be considered stable in the interval $t \leq T \approx 7$ weeks. Subsequently there arise oscillations increasing with time, which are most sharply expressed at the eastern boundary. The disturbances, which are apparently caused both by the numerical method itself and by the large number of iterations (more than 4000), arise first at the eastern boundary and then are very slowly carried westward by Rossby waves, gradually occupying the entire region. The instability associated with the fact that the level is found by an explicit formula from the integral equation of continuity,

somewhat resembles an instability, and an absolute one at that, which appears when the equation describing the propagation of Rossby waves is solved by an explicit scheme ([4]). Reducing the step by a factor of two did not change the results at $t < T$ at all and again led to oscillations at $t > T$, although with a somewhat smaller amplitude.

Fig. 2. Curves of the distribution of the meridional component of the total transport S_y at $y = (y_0 + y_1)/2$ as a function of time at the western boundary of the interior region $x = \Delta x$ (1) and at the eastern boundary $x = L - \Delta x$ (2)

Thus, the solution obtained can apparently correctly show the development of the process and even, with sufficient accuracy, its establishment toward a stationary regime. The relatively small amplitude of the transport at the western boundary is due to the fact that, first, the solution was obtained at points located one step away from the boundary, whereas the maximum value of S_y is attained in this formulation of the problem at the boundary itself ([1]), and, secondly—and this is the main point—because the calculations were carried out with a uniform step along the X-axis. Reducing the step near the western boundary, as numerical experiments showed, increases the amplitude of the current in the western boundary layer and decreases its width. As can be seen from the same figure, the current, for the selected parameter values, adjusts to the prescribed wind field in approximately 5-6 weeks (~ 1 month), which does not contradict previously obtained results ([5]).

Figure 1 gives charts of the distribution of total transports for different moments of time. The figures and calculations presented lead to the following conclusion. At first, purely drift currents arise, whose total transports deviate somewhat to the right of the wind direction; moreover, because of the variability of the Coriolis parameter, the pattern is not symmetric with respect to the mean parallel.

The divergence of the purely drift current leads to the appearance of level slopes. For some time the pattern of the distribution of total transports essentially does not change; the total transports increase, especially in the southern part of the basin, and deviate more strongly to the right of the wind direction; we obtain the pattern shown in Fig. 1 for $t = 4$ hr. After 2 days the circulation in the basin assumes the form of a gyre, with the current at the eastern boundary stronger than at the western. With time the total transport at the western boundary increases and, after approximately 1/2 week, becomes comparable with the current at the eastern boundary; moreover, both currents intensify at the walls, i.e., two boundary layers appear, both at the western and at the eastern walls. Subsequently the western boundary current continues to strengthen and is gradually pressed against the coast, while the boundary layer at the eastern coast is dissipated westward. The integral circulation obtained after approximately one month of wind action corresponds to the stationary pattern obtained in ([6]).

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Note: Figure translations are in progress. See original paper for figures.