PROPERTIES OF INTEGRALS OF TEMLYAKOV-BAVRIN TYPE

MATHEMATICS

1968

SovietRxiv

Abstract

Full Text

UDC 517.55

MATHEMATICS

V. A. GUSAKOV

PROPERTIES OF INTEGRALS OF TEMLYAKOV-BAVRIN TYPE

(Presented by Academician M. A. Lavrent' ev, 14 VI 1967)

Let D' be a bounded convex complete bicircular domain with center at the point $(0,0)$, whose boundary is twice continuously differentiable and analytically convex from the outside. As A. A. Temlyakov proved ([1,2]), the boundary of this domain is given parametrically in the form

$$|w| = r_1(\tau), \qquad |z| = r_2(\tau), \qquad 0 \leq \tau \leq 1,$$

where $r_1(\tau)$ is a function continuous on the segment $0 \leq \tau \leq 1$, and

$$r_1(0) = 0, \qquad r_1(1) < \infty, \qquad 0 < r_1'(\tau) \leq r_1(\tau)/\tau \quad (0 < \tau \leq 1),$$

$$r_2(\tau) = R_2 \exp\left[-\int_0^\tau \frac{\tau}{1-\tau}\, d\ln r_1(\tau)\right], \qquad r_2(1) = 0;$$

R_2 is a positive constant.

In the same work ([1]), integral representations of two kinds were obtained.

If the function $f(w,z)$ is holomorphic in the domain D' and continuous in the closed domain $\overline{D'}$, then for points $(w,z) \in D'$

$$f(w,z) = \frac{1}{4\pi^2 i} \int_0^1 d\tau \int_0^{2\pi} dt \int_{|\xi|=1} \frac{\xi f(r_1(\tau)\xi,\, r_2(\tau)\xi e^{-it})}{(\xi - u)^2}\, d\xi, \tag{1}$$

where

$$u = \tau \frac{w}{r_1(\tau)} + (1-\tau)\frac{z}{r_2(\tau)} e^{it}.$$

If a function $f(w,z)$, holomorphic in D', and its first partial derivatives are continuous in the closed domain $\overline{D'}$, then for points $(w,z) \in D'$

$$f(w,z)=\frac{1}{4\pi^2 i}\int_0^1 d\tau\int_0^{2\pi}dt\int_{|\xi|=1}\frac{L_1\left[f(r_1(\tau)\xi,\, r_2(\tau)\xi e^{-it})\right]}{\xi-u}\,d\xi, \qquad (2)$$

where

$$L_1[f(w,z)]\equiv f(w,z)+wf'_w(w,z)+zf'_z(w,z).$$

The integral representations (1) and (2) are known as Temlyakov integral representations of, respectively, the second and first kinds.

I. I. Bavrin ([3,4]), in the case of bounded convex complete n-circular domains D ($n\geq 2$), obtained the general Temlyakov integral representation, the general Poisson-Temlyakov integral representation, and the general Schwarz-Temlyakov formula. In the same works ([4-7]), in the case of the same class of domains, he obtained other general integral representations closely connected with the Cauchy, Poisson, and Schwarz integrals of one complex variable. We shall call the latter general integral representations, respectively, the general Temlyakov-Bavrin, Poisson-Temlyakov-Bavrin, and Schwarz-Temlyakov-Bavrin integral representations. Below we shall restrict ourselves to three of the many integral representations included in the general Temlyakov-Bavrin integral representations, and we shall take the case $n=2$ and, as the domain D, the Temlyakov domain D' indicated above. Then ([7]), if holo-

morphic in D' function $f(w,z)$ and all its partial derivatives up to and including the second order are continuous in the closed domain $\overline{D'}$, then in D'

$$f(w,z)=\frac{1}{4\pi^2 i}\int_0^1 d\varepsilon\int_0^1 d\tau\int_0^{2\pi}dt\int_{|\zeta|=1}\frac{\varphi_1(\tau,t,\zeta)}{\zeta-\varepsilon u}\,d\zeta, \qquad (3)$$

$$f(w,z)=\frac{1}{4\pi^2 i}\int_0^1 d\varepsilon\int_0^1 d\tau\int_0^{2\pi}dt\int_{|\zeta|=1}\frac{\varphi_2(\tau,t,\zeta)}{\zeta-u_1(\varepsilon)}\,d\zeta, \qquad (4)$$

$$f(w,z)=\frac{1}{4\pi^2 i}\int_0^1 d\varepsilon\int_0^1 d\tau\int_0^{2\pi}dt\int_{|\zeta|=1}\frac{\varphi_3(\tau,t,\zeta)}{\zeta-u_2(\varepsilon)}\,d\zeta, \qquad (5)$$

where

$$\varphi_1(\tau,t,\zeta)\equiv\Phi_1(r_1(\tau)\zeta,\, r_2(\tau)\zeta e^{-it}),\qquad \Phi_1(w,z)=L_1[L_1[f(w,z)]],$$

$$\varphi_2(\tau,t,\zeta)\equiv\Phi_2(r_1(\tau)\zeta,\, r_2(\tau)\zeta e^{-it}),\qquad \Phi_2(w,z)=L_{(1;1)}[L_1[f(w,z)]]$$

$$(L_{(1;1)}[f(w,z)]=f(w,z)+wf'_w(w,z)),$$

$$\varphi_3(\tau,t,\zeta)\equiv\Phi_3(r_1(\tau)\zeta,\, r_2(\tau)\zeta e^{-it}),\qquad \Phi_3(w,z)=L_{(1;2)}[L_1[f(w,z)]]$$

$$(L_{(1;2)}[f(w,z)]=f(w,z)+zf'_z(w,z)),$$

$$u_1(\varepsilon)=\tau\frac{\varepsilon w}{r_1(\tau)}+(1-\tau)\frac{z}{r_2(\tau)}e^{it},\qquad u_2(\varepsilon)=\tau\frac{w}{r_1(\tau)}+(1-\tau)\frac{\varepsilon z}{r_2(\tau)}e^{it}.$$

The integrals (3)–(5) will be called Temlyakov–Bavrin integrals of the first kind. In the present note we study integrals of Temlyakov–Bavrin type of the first kind. The theorem stated below shows that the behavior of integrals of Temlyakov–Bavrin type of the first kind has qualitative differences from the behavior of the well-known Temlyakov integral of the first kind ([8]).

Let the function $f(\tau,t,\zeta)$ be continuous jointly in the arguments (τ,t,ζ) $(0\le\tau\le1,\ 0\le t\le2\pi,\ |\zeta|=1)$ (on this topological product $f(\tau,t,\zeta)$ is defined*), be periodic with respect to the argument t with period 2π, and satisfy the Hölder–Lipschitz condition with respect to ζ:

$$|f(\tau,t,\zeta)-f(\tau,t,\zeta_0)|<K|\zeta-\zeta_0|^\alpha,\qquad 0<\alpha\le1;$$

K is a certain constant, and K and α do not depend on τ and t. We shall denote this class of functions by μ.

Let, in the expressions defining u, $u_1(\varepsilon)$, and $u_2(\varepsilon)$,

$$r_1(\tau)\equiv\tau/a,\qquad r_2(\tau)\equiv(1-\tau)/b$$

(a,b are positive constants).

Theorem. If the function $\varphi(\tau,t,\zeta)\in\mu$, then the integrals of Temlyakov–Bavrin type of the first kind

$$F=\frac{1}{4\pi^2 i}\int_0^1 d\varepsilon\int_0^1 d\tau\int_0^{2\pi}dt\int_{|\zeta|=1}\frac{\varphi(\tau,t,\zeta)}{\zeta-\varepsilon u}\,d\zeta,\tag{6}$$

$$F_1=\frac{1}{4\pi^2 i}\int_0^1 d\varepsilon\int_0^1 d\tau\int_0^{2\pi}dt\int_{|\zeta|=1}\frac{\varphi(\tau,t,\zeta)}{\zeta-u_1(\varepsilon)}\,d\zeta,\tag{7}$$

$$F_2=\frac{1}{4\pi^2 i}\int_0^1 d\varepsilon\int_0^1 d\tau\int_0^{2\pi}dt\int_{|\zeta|=1}\frac{\varphi(\tau,t,\zeta)}{\zeta-u_2(\varepsilon)}\,d\zeta\tag{8}$$

* In ([8]) $f(\tau, t, \zeta) \equiv \psi(r_1(\tau)\zeta, r_2(\tau)\zeta e^{-it})$, i.e. the function $f(\tau, t, \zeta)$ is prescribed on the boundary $\partial D'$ of the domain D'; in ([9]) the function $f(\tau, t, \zeta)$ is considered for the first time on the indicated topological product.

are:

a) (6) is a holomorphic function in the domain $A: \{a|w| + b|z| < 1\}$ and, generally speaking, a nonholomorphic function in the domain $C^2 \setminus \overline{A}$;

b) (7) is a holomorphic function in the domains $A: \{a|w| + b|z| < 1\}$ and $E_1: \{a|w| - b|z| < -1\}$ and, generally speaking, a nonholomorphic function in the domain $C^2 \setminus \overline{(A \cup E_1)}$;

c) (8) is a holomorphic function in the domains $A: \{a|w| + b|z| < 1\}$ and $E_2: \{a|w| - b|z| > 1\}$ and, generally speaking, a nonholomorphic function in the domain $C^2 \setminus \overline{(A \cup E_2)}$.

It follows from the theorem just given that the behavior of integrals of the type formed on the basis of the integral representations included in the general Temlyakov–Bavrin integral representations, generally speaking, has qualitative differences from the behavior of integrals of Temlyakov type ([8]).

Moscow Regional Pedagogical Institute
named after N. K. Krupskaya

Received
17 V 1967

CITED LITERATURE

1. A. A. Temlyakov, DAN, 120, No. 5 (1958).

2. A. A. Temlyakov, Uch. zap. Moscow Regional Pedagogical Institute named after N. K. Krupskaya, 96, 3 (1960).

3. I. I. Bavrin, DAN, 169, No. 3 (1966).

4. I. I. Bavrin, Uch. zap. Moscow Regional Pedagogical Institute named after N. K. Krupskaya, 166, 3 (1966).

5. I. I. Bavrin, DAN, 172, No. 6 (1967).

6. I. I. Bavrin, DAN, 176, No. 6 (1967).

7. I. I. Bavrin, Uch. zap. Moscow Regional Pedagogical Institute named after N. K. Krupskaya, 188, 3 (1967).

8. L. A. Aizenberg, DAN, 120, No. 5 (1958).

9. V. I. Bogdanov, G. L. Lukankin, DAN, 176, No. 1 (1967).

Note: Figure translations are in progress. See original paper for figures.