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Abstract

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MATHEMATICAL PHYSICS

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ON DIFFRACTION EFFECTS IN MEASUREMENTS OF THE VELOCITY AND ABSORPTION OF ULTRASOUND

(Presented by Academician I. M. Vinogradov, 19 XII 1967)

1. In measurements of the velocity v and absorption coefficient α of ultrasound in arrangements with plane radiators, one-dimensional methods are usually used, based on the assumption that the sound field has the form of plane waves traveling along the z -axis, normal to the plane of the radiator $z = 0$. However, for accurate measurements of v and α , corrections for wave diffraction must be introduced into these methods (¹⁻⁴). Here a method for calculating diffraction is given (Sec. 2), with the aid of which the range of applicability of these corrections and their accuracy are investigated.

2. Method of packets of normal waves. The field produced for $z > 0$ by a given distribution of normal velocities $V(r, \theta, z = 0) \times \exp i\omega t$ on the radiator will be described by the velocity potential $\varphi(r, \theta, z) \exp i\omega t$, where r, θ , and z are cylindrical coordinates, and ω is the sound frequency. Suppose that the response amplitude $\bar{p}(z)$ of the ultrasonic transducer to a pressure wave with amplitude $p = i\omega\rho_0\varphi(r, \theta, z)$, incident on it after reflection from the plane of the reflector $z = l$, is equal to

$$\bar{p}(l) = D \int_F p(r, \theta, z = 2l) V^*(r, \theta, z = 0) r dr d\theta, \quad (1)$$

where F is the surface of the radiator, D is a proportionality coefficient, and the asterisk denotes the complex conjugate quantity. The justification of (1) will be given elsewhere. Neglecting the side walls of the apparatus and assuming that V does not depend on θ , we expand φ in a continuous spectrum of normal waves traveling along the z -axis. Then, in the case of a traveling-wave regime,

$$\bar{p}(z) = \omega\rho_0 \int_0^\infty \Phi(\chi) e^{-i\gamma z} d\chi, \quad (2)$$

where $\gamma = \sqrt{k^2 - \chi}$ is the wave number of the normal wave determined by the parameter χ ; $k = \beta - i\alpha$ is the wave number of a wave with a homogeneous front ($\chi = 0$), $\beta = \omega/v = 2\pi/\lambda$; λ is the wavelength for $\chi = 0$, and

$$\Phi(\chi) = \frac{1}{a^2\gamma} \left[\int_0^\infty V(r) J_0(\sqrt{\chi} r) r dr \right]^2 \quad (3)$$

is the spectral function of the apparatus. Below, finite radiators are considered; for them $V \neq 0$ only for $r \leq a$. We set $D = 1/\pi a^2$, so that for piston radiators ($V = \text{const}$ for $r \leq a$) \bar{p} is equal to the mean pressure $\langle p \rangle$ on the surface of the radiator. We approximate $\Phi(\chi)$ for $0 < \chi < \beta^2$ by a Chebyshev function

$$\Phi^N(\chi) = \sum_{n=0}^N \Phi_n(\chi) = \sum_{n=0}^N \left\{ \frac{C_n^{(1)}}{(\chi - \chi'_n) - i\chi''_n} + \frac{C_n^{(2)}}{(\chi - \chi'_n) + i\chi''_n} \right\}; \quad (4)$$

$\chi_n = \chi'_n \pm i\chi''_n$ —the poles of Φ_n . We shall call the component Φ_n the n -th spectral line of the oscillation form V . It has width $2\chi''_n$ and center χ'_n . Its even part has amplitude $B_n^s = (C_n^{(1)} - C_n^{(2)})/2$, and its odd part $B_n^{as} = (C_n^{(1)} + C_n^{(2)})/2$. Substituting (4) into (2), we obtain

$$\bar{p}(z) \cong \sum_{n=0}^N \bar{p}_n(z), \quad (5)$$

where the response component \bar{p}_n is determined through Φ_n by means of integral (2). Assuming that the quasioptical conditions are satisfied, $n = 0, 1, 2, \dots, N$,

$$\sqrt{k^2 - (\chi'_n + \delta\chi)} \cong \sqrt{k^2 - \chi'_n} - \frac{\delta\chi}{2\sqrt{k^2 - \chi'_n}}, \quad (6)$$

we obtain, after integration,

$$\bar{p}_n = \rho_0 \omega e^{-i\gamma_n z} \left\{ C_n^{(2)} e^{-x_n} + \frac{i}{\pi} \left[C_n^{(2)} e^{-x_n} \bar{\text{Ei}}(x_n) + C_n^{(1)} e^{x_n} \text{Ei}(-x_n) \right] \right\}, \quad (7)$$

where $\bar{\text{Ei}}(x)$ and $\text{Ei}(-x)$ are exponential integral functions, and

$$x_n = \chi''_n z / 2\gamma_n,$$

$$\gamma_n = \beta_n - i\alpha_n = \sqrt{k^2 - \chi'_n}. \quad (8)$$

Fig. 1

Fig. 1

Figure 1: Fig. 1

It follows from (7) that each spectral line gives rise to a packet of normal waves, represented in the quasioptical approximation by a single wave. The latter undergoes diffraction attenuation, described by the term in braces in (7). Introducing the diffraction-decay function $A(x_n) = 20 \log\{|\bar{p}_n(0)|/|\bar{p}_n(x_n)|\}$, we present it for an even line by curve A in Fig. 1. For small x_n , the first term in (7) dominates it, with exponential-decay coefficient $\alpha_d^n = \chi_n''/2\beta_n$ (curve); for large x_n , the term in square brackets of (7) dominates (curve), representing a spherical wave*. For $x_n < 2 \div 3$, $A(x_n)$ may be approximated by a linear dependence (curve) with coefficient $\alpha_d^n \cong 0.5\alpha_d^n$, i.e.

$$\bar{p}_n(z) \cong i\rho_0\omega\pi B_n^s e^{-(\alpha_n - \bar{\alpha}_d^n)z} e^{-i\beta_n z}. \quad (9)$$

For $x_n > 2 \div 3$

$$\bar{p}_n(z) \cong 2\rho_0\omega B_n^s \frac{e^{-\alpha_n z - i\beta_n z}}{\alpha_d^n z} \left[1 + \frac{2!}{(\alpha_d^n)^2 z^2} + \frac{4!}{(\alpha_d^n)^4 z^4} + \dots \right]. \quad (10)$$

The contribution of the odd part of the spectral line, for the same amplitudes, is represented by curve E ; it decays rapidly as z increases.

* The ratio of the second term to the first is shown by curve A .

3. Let us consider radiators with the following properties. 1) Small reactive power. 2) Among $\Phi_n(\chi)$, $0 < \chi_n' < \beta^2$, in the interval $(\chi_1'' - \tilde{\chi}, \chi_1'' + \tilde{\chi})$, $|\tilde{\chi}| < 1$, there exists only one even line $n = 1$; all the other even lines belong to type a or b; for type-a lines $\chi_n'' < \chi_1''$ and $|B_n| \ll |B_1|$, and for type-b lines $\chi_n'' > \chi_1''$. (In accordance with this, $\bar{p}(z)$ consists of the sum of packets of type a, type b, and the principal packet $n = 1$.) 3) The odd lines have amplitudes not exceeding $|B_1|$.

The sum of the responses of packets of type b attenuates with increasing z , becoming less than \bar{p}_1 by a factor K at $z \geq z_1$, where z_1 is determined from the condition

$$K \sum_n^{(\delta)} |B_n^s| e^{-(a_n + \alpha_d^n)z_1} = |B_1^s| e^{-(a_1 + \alpha_d^1)z_1}. \quad (11)$$

For $z > z_2'$, where z_2' is determined from an analogous condition in which the summation is restricted to packets of type a, the response \bar{p}_1 becomes K times smaller than the sum of the responses of packets of type a. If $z_1 < z_2'$, then in the interval (z_1, z_2') the response \bar{p}_1 dominates in \bar{p} . In this case, if $z_1 < z_2$,

where $z_2 = (2 \div 3)2\beta/\chi_1''$, then in the interval (z_1, z_2) for $z_2 < z_2'$ and in (z_1, z_2') for $z_2 > z_2'$, the diffraction corrections introduced into one-dimensional methods will be

$$\Delta v = \frac{\chi_1' \lambda^2}{8\pi^2} v + O(K^{-1}), \quad \Delta \alpha = \bar{\alpha}_d^{-1} + O(K^{-1}), \quad \bar{\alpha}_d^{-1} \cong \frac{\chi_1' \lambda}{8\pi}. \quad (12)$$

For large K they are close to the corrections given in works ⁽¹⁻³⁾. The class of radiators satisfying conditions 1-3 of this item, for which z_1 is smaller than z_2 and z_2' , and the terms $O(K^{-1})$ in (12) are an order of magnitude smaller than the principal terms, will be called the class suitable for measurements by the method ⁽¹⁻³⁾, or, for short, the class Π_z^K . The regions $z < z_1$ and $z > z_2'$, apparently, are of little use for determining v and α , since in them $\bar{p}(z)$ is sensitive to the details of the function $V(r, \theta)$. An experimental criterion for the suitability of the corrections (12) is the closeness of the dependence $\log |p(z)|$ to a linear one; χ_1' and χ_1'' must be determined from experiment. Using the method of item 2, one can prove the following two theorems.

Theorem 1. *For circular piston radiators of radius a , the condition $|ka| \gg 1$ is sufficient for them to belong to the class Π_z^K .*

Theorem 2. *The corrections Δv and $\Delta \alpha$ for circular radiators of given radius a cannot be smaller than the values*

$$\Delta \alpha_{\text{por}} \cong 0.1\lambda/a^2, \quad \Delta v_{\text{por}} \cong v\lambda^2/31.5a^2, \quad (13)$$

attaining them for piston radiators (the extremality property).

4. $\Delta \alpha_{\text{por}}$ coincides with the correction obtained in work ⁽⁴⁾ by calculating the Rayleigh integral for a circular piston radiator. Let us compute this case by the method of item 2. For such a radiator Φ takes the form

$$\Phi(\chi) = a^2 J_1^2(\sqrt{\chi}) / \bar{\chi} \sqrt{(ka)^2 - \bar{\chi}}, \quad \bar{\chi} = \chi a^2. \quad (14)$$

For $|ka| \gg 1$, approximation of (14) by the function (4), when N is equal to the number of maxima of the function (14) in the interval $0 < \chi < \beta^2$, gives

$$n = 0: \quad C_0 = C_0^{(1)} = C_0^{(2)} = 0.1/k, \quad \bar{\chi}'_0 = \bar{\chi}''_0 = 1,$$

$$n = 1: \quad C_1 = C_1^{(1)} = -C_1^{(2)} = 0.32i/k, \quad \bar{\chi}'_1 = \bar{\chi}''_1 = 2.5.$$

The remaining lines $n = 2, 3, 4, \dots$ have $|C_n| < |C_2| = 0.036/|k|$, while $\chi'_n = 30, 70, 140, \dots$ and $\chi''_n = 8, 12, 20, \dots$. The dependence $A(x)$, calculated by formulas (5), (7) for the parameters Φ^N given above, is represented by curve D in Fig. 1. To graphical accuracy it coincides with $A(S)$ ($S = \lambda z/a^2$) of work ⁽⁴⁾;

the scale $S(x = 0.2S)$ is given at the top of Fig. 1. In the interval (S_1, S_2) the slope $A(S)$ determines $\Delta\alpha_{\text{por}}$ from (13). Oscillations at $S < S_1$ are caused by spatial beats of packets of type b with $n = 1$

and with one another. By the choice of $V(r)$ they can be weakened, but then, according to Theorem 2, $\Delta\alpha$ and Δv will become larger (13). Piston radiators do not have lines of type a, which cause beats for $S > S'_2$ ($z > z'_2$).

5. **In the case of interferometers** we shall restrict ourselves to piston-like radiators, when the input impedance of the transducer $Z'_1 = j(Z_m)$, where $Z_m = \langle p \rangle / \langle V \rangle$ is the acoustic impedance of the medium, is calculated by formula (1.46)⁽⁵⁾, derived for an infinite piston transducer. Applying the method of Sec. 2, we obtain

$$Z_m = \left\{ \bar{p}_0 + 2 \sum_{n=1}^{\infty} \bar{p}_m(l) \right\} \simeq \sum_{n=0}^N \left\{ \bar{p}_{0,n} + 2 \sum_{m=1}^{\infty} \bar{p}_{m,n} \right\}, \quad (15)$$

where $\bar{p}_{m,n}$ is the response component of the wave packet of line n that has undergone m reflections from the reflector. If, for the wave packets dominating in (15), $x < 2 \div 3$, then

$$Z_m \simeq i\rho_0\omega\pi \sum_{n=0}^N B_n^2 \text{ctg} [\bar{\gamma}_n l + \text{const}], \quad \bar{\gamma}_n = \beta_n - i(\alpha_n + \bar{\alpha}_n^d). \quad (16)$$

Imposing on $\Phi(x)$ conditions similar to those of Sec. 3, we single out an interval of the l -axis of the response curve $Z_1(l)$, (l_1, l_2) , in which the fundamental packet $n = 1$ dominates in (16) and the diffraction corrections (12), (13) are applicable. (Radiators of class Π_l^K .) Piston radiators belong to this class for $ka \gg 1$. For $l < l_1$, additional systems of peaks (satellites), called diffraction peaks in⁽⁶⁾, are possible on the response curve. They should not be confused with satellites that remain at large l ⁽¹⁻³⁾.

According to current views, the interferometer considered here is an open resonator, and the diffraction attenuation is caused by radiation losses through its lateral surface.

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