ON THE THEORY OF HEATING IN HYPERSONIC FLOW

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Abstract

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ON THE THEORY OF HEATING IN HYPERSONIC FLOW

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In earlier works devoted to the theory of aerodynamic heating of blunt bodies, the convective and radiative components were calculated independently. Convective heating was determined by the parameters of a boundary layer calculated without allowance for radiation. In calculating radiative heating it was assumed that the temperature and pressure in the compressed layer do not depend on the coordinates. It was shown that, as the velocity of the oncoming flow increases, the radiative component grows much faster than the convective one and, beginning with the second cosmic velocity, becomes dominant. However, precisely under these conditions the approximations used in the earlier works become unacceptable. Radiation fluxes become a powerful factor in energy transfer. This must lead to a substantial dependence of the temperature on the coordinates throughout the compressed layer. The possibility of isolating a boundary layer becomes doubtful.

There are attempts in the literature to solve the combined problem, including allowance for the influence of radiation transfer on the gas-dynamic parameters and on aerodynamic heating. In most cases these works used approximations that do not ensure even the correct order of the quantities calculated (the gray-gas approximation, the assumption of complete transparency of the compressed layer). An exception is work ([1]), in which flow and heating during motion in the Earth' s atmosphere are also considered jointly, and an attempt is made to take the real spectrum of air into account. However, the authors of ([1]) were forced to neglect energy transfer in spectral lines, although, referring to ([2]), they did note the importance of this process.

The present work is likewise an attempt to solve the combined problem of the flow of a supersonic stream of air past a blunt body. The flow in the neighborhood of the critical line is considered. The boundary-layer approximation is not used. Viscosity, thermal conductivity, and energy transfer by radiation are taken into account. The real spectrum of air, including spectral lines, is considered. The presence of local thermodynamic equilibrium is assumed.

In spherical coordinates (r, θ, φ), putting $\theta \to 0$, we obtain the system of equations

$$r\frac{d}{dr}(\rho v) + 2\rho(u + v) = 0; \tag{1}$$

$$\rho v r\frac{du}{dr} + \rho u^2 + \rho u v + \left(\frac{\partial^2 p}{\partial \theta^2}\right)_{\theta=0} = r\frac{d}{dr}\left(\mu\frac{du}{dr}\right); \tag{2}$$

$$\frac{dp}{dr} + \rho v\frac{dv}{dr} = 0; \tag{3}$$

$$\rho v\frac{d}{dr}\left(h + \frac{v^2}{2}\right) = \frac{d}{dr}\left(\frac{\mu}{\mathrm{Pr}}\frac{dh}{dr}\right) - Q(r); \tag{4}$$

$$r\frac{d}{dr}\left(\frac{\partial^2 p}{\partial \theta^2}\right)_{\theta=0} = 2\rho u^2, \tag{5}$$

where h is enthalpy; p is pressure; v is the radial and u' the tangential component of velocity; $u = (\partial u'/\partial\theta)_{\theta=0}$. The values of the density $\rho(T, p)$ were taken from (3); the viscosity $\mu(T, p)$ and the Prandtl number $\mathrm{Pr}(T, p)$, from (4). For what follows it proved convenient first to obtain from these data $\rho(h, p)$, $\mu(h, p)$, and $\mathrm{Pr}(h, p)$. Equation (5) was obtained by differentiating with respect to θ the equation for transport of the radial component of momentum. In doing so, an expansion was used in powers of a small quantity—the ratio of the densities before and after the shock.

The quantity $Q(r)$ is the divergence of the radiative-energy flux density. In computing $Q(r)$, the shock layer was replaced by a plane layer of thickness equal to the shock stand-off distance δ. Such a replacement introduces no significant error, since $\delta \ll R$, where R is the radius of curvature of the body being flowed around at the critical point. In this approximation

$$Q(r) = \int_0^\infty \varepsilon_\nu(r)\,d\nu - \frac{1}{2}\int_0^\infty d\nu \int_R^{R+\delta} \varepsilon_\nu(r')K_\nu(r)\,\mathrm{Ei}\left(\left|\int_{r'}^{r} K_\nu(r'')\,dr''\right|\right)dr', \tag{6}$$

where ε_ν is the emissivity; K_ν is the absorption coefficient. Under conditions of local thermodynamic equilibrium, $\varepsilon_\nu = B_\nu K_\nu$, where B_ν is the Planck function.

The boundary conditions at $r = R + \delta$ are obtained from the shock relations; at $r = R$ the enthalpy corresponding to the body temperature is prescribed, and the conditions $u = 0$, $v = 0$ are also required. It is assumed that the medium for $r < R$ and for $r > R + \delta$ neither emits nor reflects the radiation flux.

Fig. 1

Figure 1: Fig. 1

Fig. 1

The system (1)–(6) was solved as follows. First, specifying certain initial values of u and h (and hence of ρ), equations (1), (3), and (5) were solved for v, p, and $(\partial^2 p/\partial\theta^2)_{\theta=0}$. The boundary conditions then yielded δ. These values were substituted into (2) and (4), which, because of their nonlinearity, were solved by iteration. In this process, some initial $Q(r)$ was used. As a result, new u and h were obtained, which were used for a repeated solution of (1), (3), and (5). The parameter profiles obtained after convergence had been reached were used in (6) to compute the next approximation to $Q(r)$, after which the entire procedure was repeated until full convergence was achieved. In the final result, the shock-wave stand-off distance, the profiles of velocity, temperature, and pressure in the compressed layer, and the convective ($\dot{q}_c$) and radiative ($\dot{q}_r$) components of heating were determined ($\dot{q}_c$ was found from the value of $(dT/dr)_{r=R}$). It should be noted that direct use of (6) to compute $Q(r)$ does not seem possible. The tables of $K_\nu(p,T)$ available in the literature do not include the contribution of spectral lines, which, for the parameters of interest to us, is very substantial ([2]). The tables at our disposal have the smallest step in frequency and give the total absorption coefficient, including spectral lines. However, only strongly broadened lines are accounted for with sufficient accuracy. Compiling tables that would give a sufficiently accurate description of the entire ensemble of lines is impossible, since in that case the volume of the tables would increase to unrealistic dimensions. In this connection, in computing $Q(r)$, expression (6) was transformed in accordance with an approximate method for allowing for the energy transfer of spectral li-

proposed in ([5]). We note that this method takes into account the dependence on the coordinates of both the intensity and the shape of the spectral lines. The same method was used in ([6]). In the present work we used the computational program compiled in carrying out ([6]).

Solutions were obtained for a number of combinations of the free-stream velocity V_∞, the pressure ahead of the shock p_∞, and the radius of curvature of the body at the critical point R. As an example, Fig. 1 gives temperature profiles for the case $p_\infty = 0.27 \cdot 10^{-4}$ atm, $V_\infty = 18$ km/sec, $R = 1$ m. The dashed line corresponds to the temperature profile obtained without allowance for radiation, the solid line–with allowance for the latter. $\dot{q}_c/\dot{q}_c^0 = 0.8$ and $\dot{q}_r/\dot{q}_r^0 = 0.4$, where $\dot{q}_c^0$ was obtained in solving the problem without allowance for radiation and is very close to the value that follows from the commonly used Fay–Riddell formula; $\dot{q}_r^0$ was computed in the approximation of a homogeneous compressed layer. It is appropriate to note that, under the conditions used in Fig. 1, the contribution of spectral lines to the resulting flux is one and a half times greater than the contribution of the remaining radiative processes.

Table 1 gives the values of the radiative $\dot{q}_r$ and convective $\dot{q}_c$ components of the heat flux, as well as the values $\dot{q}_r^0$ and $\dot{q}_c^0$ for $p_\infty = 10^{-4}$ atm, $R = 1$ m, and two values of the free-stream velocity.

Table 1

V_∞, km/sec	$\dot{q}_r$, kW/cm^2	$\dot{q}_c$, kW/cm^2	$\dot{q}_r^0$, kW/cm^2	$\dot{q}_c^0$, kW/cm^2
14	0.89	0.37	1.65	0.53
12	0.34	0.27	0.5	0.33

Analysis of the results makes it possible to draw several preliminary conclusions. Allowance for radiation transfer practically does not change the pressure in the compressed layer, but it substantially lowers the temperature and correspondingly increases the density. The latter leads to some decrease in the stand-off distance (5-20%).

It is evident that a change in the temperature profile must affect both the radiative and the convective components of heating. The radiative component decreases; moreover, because of the sharp dependence of the emissivity on temperature, the decrease may be very substantial even when the change in temperature is small.

In the cases we considered, cooling of the compressed layer led to a decrease in the convective component of heating. However, there may be conditions under which, as a result of absorption of radiation by the relatively cold layers near the body, $(dT/dr)_{r=R}$ increases, which will lead to an increase in $\dot{q}_c$. The effects noted above increase with increasing temperature at the shock and become practically significant beginning with the second cosmic velocity. We note that, as the temperature at the shock increases and, consequently, throughout the compressed layer, the role of thermal conductivity also increases. Consideration of aerodynamic heating at high V_∞ while neglecting thermal conductivity and viscosity is apparently inexpedient. Let us consider the influence of the radius of curvature of the body. As R increases, and hence δ, the time of motion of an element of gas volume from the shock to the body increases. Consequently, an increase in R intensifies the effects associated with radiative cooling. It is appropriate to note that allowance for radiation transfer leads to a change in the regularities established for convective and radiative heating when they are considered separately. Thus, $\dot{q}_c$ decreases faster than $1/\sqrt{R}$. Conversely, for the radiative component, allowance for energy transfer by radiation leads to a substantial slowing of the growth of $\dot{q}_r$ with increasing R. At large R, saturation and even a decrease of $\dot{q}_r$ with increasing R are possible. The latter may occur in the case when, with increasing R, the effect of ra-

radiative cooling will exceed the increase in the radiating capacity of the compressed layer caused by the increase in R. Under these conditions, the radiation of the hot layers located near the shock will be absorbed mainly in the central part of the compressed layer and will not reach the body. A change in the ratio between $\dot{q}_c$ and $\dot{q}_r$, as well as in the dependence of these quantities on R, requires reconsideration of the question of choosing the optimal shape of the body being flowed around.

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Note: Figure translations are in progress. See original paper for figures.