RADIATIVE EFFECTS IN EXPERIMENTS WITH COLLIDING ELECTRON-POSITRON BEAMS

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Abstract

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RADIATIVE EFFECTS IN EXPERIMENTS WITH COLLIDING ELECTRON-POSITRON BEAMS

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1. In connection with experiments currently being carried out with colliding electron-positron beams ([1,2]), the study of photon-emission processes in electron–positron collisions is of considerable interest.

The formulation of the problem and a concrete examination of this range of questions for electron–electron collisions were carried out in papers ([4,6]). In the present article the method used in those papers is employed. Expressions are found for the cross section of electron–positron scattering with emission of a photon of arbitrary energy into narrow cones along the directions of motion of the particles. The cross section for emission of soft quanta is obtained for arbitrary electron scattering angles. With logarithmic accuracy, expressions are obtained for the radiative corrections to the electron–positron scattering cross section.

2. Let us consider the process of emission of a photon with energy ω along the direction of the momentum of the initial electron $\mathbf{p}_1$ in the case where the angular dimensions of the photon detector are $2/\gamma < 2\vartheta_0 \ll 1$ and the scattering angles ϑ are large ($\sin\vartheta \gg 1/\gamma$).

We adopt the following notation: $\mathbf{p}_1, \mathbf{p}_3 (\mathbf{p}_2, \mathbf{p}_4)$ are the momenta of the initial and final electron (positron); $\mathbf{k}$ is the photon momentum; $\gamma = E/m$. We regard the final particles as ultrarelativistic ($1 - \omega/E \gg 1/\gamma$). Since integration over the photon emission angle will be carried out with accuracy up to terms of order $\vartheta_0^2, 1/\gamma^2$, using formulas (2.1)-(2.5) of paper ([3]), with the corresponding substitutions, and retaining in them the principal terms giving an integral contribution with the indicated accuracy, we obtain

$$d\sigma_1 = \frac{\alpha^3}{8\pi^2}\frac{d\xi\, d\Omega_k\, d\Omega_3}{(1-c)^2}\left\{1 + \frac{(1+c)^4 + (1-c)^4(1-\xi)^4}{f^4(\xi)}\right\}\times$$
$$\times\left[\left(1 + \frac{1}{(1-\xi)^2}\right)\frac{1}{(kp_1)} - \frac{m^2\xi}{(1-\xi)(kp_1)^2}\right], \quad (1)$$

where

$$\xi = \omega/E; \qquad c = \cos\vartheta; \qquad f(\xi) = 2 - \xi(1-c). \tag{2}$$

In the case of soft photons and photon emission angles $\vartheta_k \leq 1/\gamma$, formula (1) behaves analogously to formula (1) of paper ([4]).

As was noted in paper ([4]), if one particle is scattered through an angle ϑ, then for a given energy ω of the photon emitted along $\mathbf{p}_1$, the second particle emerges at an angle χ, where, to the adopted accuracy,

$$\cos\chi = 1 - \frac{2(1+c)}{2 - \xi(2-\xi)(1-c)}. \tag{3}$$

Therefore the expression for the cross section for photon emission along the direction of the momentum of the initial positron is obtained from formula (1) by the substitution

$$c \to 1 - \frac{2(1-c)}{2 - \xi(2-\xi)(1+c)}, \qquad p_1 \to p_2, \tag{4}$$

moreover, this substitution must be made throughout formula (1), including in the volume element $d\Omega_3$. Then we obtain

$$d\sigma_2 = \frac{\alpha^3}{8\pi^2} \frac{d\Omega_3\, d\xi\, d\Omega_k}{(1-c)^2} \frac{(1-\xi)^2}{(kp_2)} \left[1 + \frac{1}{(1-\xi)^2} - \frac{m^2\xi}{(1-\xi)(kp_2)}\right] \times$$
$$\times \left\{1 + \frac{(1-c)^4 + (1+c)^4(1-\xi)^4}{f_1^4(\xi)}\right\}, \tag{5}$$

where $f_1(\xi) = 2 - \xi(1+c)$.

Integrating over the photon emission angles, we obtain

$$d\sigma_1 = \frac{\alpha r_0^2}{4\pi\gamma^2} \frac{d\xi}{\xi} \frac{d\Omega_3}{(1-c)^2} \left[1 + \frac{(1+c)^4 + (1-c)^4(1-\xi)^4}{f^4(\xi)}\right] \times$$
$$\times \left[\left(1 + \frac{1}{(1-\xi)^2}\right) \ln(1+n^2) - \frac{2n^2}{(1+n^2)(1+\xi)}\right], \tag{6}$$

where $n = \gamma\vartheta_0$.

The expression for $d\sigma_2$ is obtained in an entirely analogous way.

The expression for the cross section for photon emission along the direction of the momentum p_3 of the final electron has a form analogous to that obtained in work ([4]):

$$d\sigma_3 = d\sigma_0 \frac{\alpha}{2\pi}\frac{d\xi}{\xi}\left[(1+(1-\xi)^2)\ln\left(1+n^2(1-\xi)^2\right) - \frac{2n^2(1-\xi)^3}{1+n^2(1-\xi)^2}\right], \qquad (7)$$

where $d\sigma_0$ is the Bhabha cross section.

3. The cross section for the emission of soft photons ($\xi \ll 1$) for electron scattering angles $\vartheta > 1/\gamma$ into a small angle $\sigma_0 \ll 1$ along the direction of the momentum of the initial electron is calculated with accuracy up to terms ϑ_0^2, $1/\gamma^2$. In contrast to radiation in electron-electron collision, in which the cross section for the emission of soft quanta in scattering through an angle $0(\pi)$ is equal to 0, we shall distinguish two cases: 1) $1/\gamma < \vartheta \leq \pi/2$, 2) $\pi/2 < \vartheta \leq \pi$. The cross section for the emission of soft quanta in the first case coincides with the radiation cross section in electron-electron scattering (formulas (7), (8) of work ([4])), where $d\sigma_0$ is now the Bhabha cross section.

The radiation cross section for the second case has the form

$$d\sigma = d\sigma_0 \frac{\alpha}{2\pi}\frac{d\omega}{\omega}\left\{\frac{z'}{\sqrt{{z'}^2+(1-\mu^2)}} - \frac{2n^2}{1+n^2} - 1 + 4\ln(1+n^2)\right.$$

$$-\frac{1+2{x'}^2}{x'\sqrt{1+{x'}^2}}\ln\frac{(1+n^2)(x'+\sqrt{1+{x'}^2})^3}{2\left[2x'(1+{x'}^2)-\gamma^2x'(1-\beta\mu)+\sqrt{1+{x'}^2}\,\gamma\beta\sqrt{{z'}^2+(1-\mu^2)}\right]}$$

$$\left.+4\left[1+\frac{{x'}^4}{2\gamma^2(\gamma^2-{x'}^2)}\right]\ln\frac{4{x'}^2}{\gamma\beta\sqrt{{z'}^2+(1-\mu^2)}+z'\gamma-\mu}\right\}. \qquad (8)$$

Here

$$4m^2{x'}^2 = 4p^2\cos^2\vartheta/2, \qquad z' = \gamma(\mu-\beta)+\frac{2{x'}^2}{\gamma\beta}; \qquad \mu = \cos\vartheta_0. \qquad (9)$$

Analogously to article ([4]), for the case $\sin\vartheta \gg 1/\gamma$ we obtain the expression for the cross section

$$d\sigma = d\sigma_0 \frac{\alpha}{\pi}\frac{d\omega}{\omega}\left[\ln(1+n^2) - \frac{n^2}{1+n^2}\right]. \qquad (10)$$

Here the probability of radiation does not depend on the scattering angle. This is due to the fact that, at large scattering angles, only the initial electron radiates into a small angle. The cross section for the emission of soft quanta is maximal at $\vartheta = \pi$ and is equal to four times the cross section expressed by formula (10), since, for scattering through an angle π, both the initial electron and the final positron radiate into a small angle along the momentum of the initial electron, and their radiation is coherent.

4. Let us turn to the consideration of radiative corrections to electron-positron scattering at large angles. For the modern formulation of experiments with colliding electron-electron beams, the question of radiative corrections was considered in papers ([5],[6]), in which two possible definitions of a scattering event and two formulations of the problem were given. In the case of electron-positron collisions, both formulations are meaningful. In the calculation we retain large logarithmic terms of the type $\ln\gamma$, $\ln\Delta\vartheta$, $\ln\Delta\psi$ *, while terms of order unity will be systematically neglected. The expression for the vacuum contributions and the contributions from the emission of soft photons in the e^6 approximation of perturbation theory were obtained in papers ([7],[8]) and may be written in the form

$$d\sigma_{el} + d\sigma_{soft} = d\sigma_0 \left\{ 1 - \frac{4\alpha}{\pi} \left[-\frac{11}{6} \ln\gamma + \frac{1}{2} \left(\ln\frac{4\gamma^2(1-c)}{(1+c)} - 1 \right) \ln\frac{1+c}{2\xi_0^2} \right] \right\}. \tag{11}$$

The calculations are carried out analogously to paper ([6]).
With the indicated accuracy, the cross sections for electron-positron scattering, taking into account radiative corrections of order e^2, turn out to be equal for both formulations of the problem and have the form:

$$d\sigma = d\sigma_0(1+\delta), \tag{12}$$

$$\delta = \frac{2\alpha}{\pi} \left\{ \ln\gamma \left[\frac{11}{3} - 2\eta + \frac{\eta^2}{2} + 2\ln\frac{2\eta}{1+c} \right] + \ln^2(\xi_0\eta) + \ln\xi_0 \left[2\ln\gamma - 2\eta + \frac{\eta^2}{2} - 1 - 2\ln\frac{(\Delta\vartheta + \Delta\psi)(1+c)}{8(1-c)} \right] \right\}. \tag{13}$$

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CITED LITERATURE

[1] V. L. Auslender, G. I. Budker et al., Proc. Intern. Symposium on Electron and Positron Storage Rings, Saclay, 1966.

[2] I. E. Augustin, R. A. Beck et al., ibid.
[3] U. N. Baier, S. A. Kheifets, Nucl. Phys., **47**, 313 (1963).
[4] V. N. Baier, V. S. Fadin, V. A. Khoze, DAN, **174**, No. 2 (1967).
[5] S. Tsai, Proc. Intern. Symposium on Electron and Photon Interaction at High Energies, Hamburg, **2**, 1965, p. 387.
[6] V. N. Baier, V. S. Fadin, V. A. Khoze, Yadern. fiz., **6**, No. 3 (1967).
[7] R. Polovin, ZhETF, **31**, 449 (1956).
[8] M. Readhead, Proc. Roy. Soc., **A220**, 219 (1953).

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* The notation in this section is taken from paper ([6]).

Note: Figure translations are in progress. See original paper for figures.