# DYNAMIC STABILITY AND VIBRATIONS OF THREE-LAYER PANELS IN A SUPERSONIC GAS FLOW

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## Abstract

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# DYNAMIC STABILITY AND VIBRATIONS OF THREE-LAYER PANELS IN A SUPERSONIC GAS FLOW

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§ 1. The system of differential equations of small vibrations of a three-layer plate, under certain assumptions, can be reduced to a single partial differential equation for the displacement function ($^1$). In ($^2$) a number of exact solutions were given for boundary-value problems of natural vibrations of three-layer beams in vacuum.

For the case of cylindrical bending of a plate (beam) in a gas flow, the indicated equation takes the form

$$\left(1-\vartheta h^2\beta^{-1}\frac{\partial^2}{\partial x^2}\right)\frac{\partial^4\chi}{\partial x^4}-N_x\left(1-h^2\beta^{-1}\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2\chi}{\partial x^2}+\frac{\Omega}{D}\frac{\partial^2}{\partial t^2}\left(1-h^2\beta^{-1}\frac{\partial^2}{\partial x^2}\right)\chi=\frac{q}{D}. \tag{1,1}$$

Here $\chi(x,t)$ is the displacement function related to the deflection $w(x,t)$ by the relation

$$w(x,t)=\left(1-h^2\beta^{-1}\frac{\partial^2}{\partial x^2}\right)\chi(x,t); \tag{1,2}$$

$q(x,t)$ is the aerodynamic load on the panel; $N_x$ is the longitudinal force in the initial surface; $x$ is the coordinate in the direction of the gas flow impinging on the three-layer panel; $t$ is time; the constants $\vartheta, h, \beta, \Omega, D$ characterize, respectively, the bending stiffness of the face layers, the thickness of the panel core, the shear stiffness of the core, the mass per unit area, and the bending stiffness of the panel ($^1$).

We assume that a supersonic gas flow impinges on the plate from one side, and that the aerodynamic load $q$ may be computed in the linear approximation of piston theory,

$$q = -\frac{\chi p_0}{a_0}\left(\frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x}\right), \tag{1,3}$$

where $p_0, a_0, V$ are, respectively, the pressure, speed of sound, and velocity of the undisturbed gas flow, $\chi = c_p/c_v$.

Represent a particular solution of (1,1) in the form

$$\chi(x,t) = \chi(x)e^{i\omega t}, \tag{1,4}$$

where $\chi(x)$ is a complex function of the real variable $x$; $\omega$ is the complex vibration frequency of the panel.

Substituting (1,4) into (1,1) and taking (1,3) into account, we obtain, passing to dimensionless parameters and to the dimensionless variable $x/l$, and retaining for it the same notation ($l$ is the panel length in the flow direction),

$$a_6\frac{d^6\chi}{dx^6} + a_4\frac{d^4\chi}{dx^4} + a_3\frac{d^3\chi}{dx^3} + a_2\frac{d^2\chi}{dx^2} + a_1\frac{d\chi}{dx} - \lambda\chi = 0, \tag{1,5}$$

where

$$\begin{gathered} a_6 = \vartheta k, \qquad a_4 = -(1+kn_x), \qquad a_3 = kp_*, \\ a_2 = -(V_*^2\omega_*^2 - ip_*\omega_*)k + n_x, \qquad a_1 = -p_*, \\ \lambda = V_*^2\omega_*^2 - ip_*\omega_*; \end{gathered} \tag{1,6}$$

$$\omega_* = \omega l/V, \qquad p_* = \frac{\chi p_0 l^3}{D}\mathrm{M}, \qquad V_* = lV\sqrt{\Omega/D}, \tag{1.7}$$

$$n_x = N_x l^2/D, \qquad k = h^2 l^{-2}\beta^{-1}, \qquad \mathrm{M} = V/a_0.$$

If the dimensionless parameters are taken in the form

$$\omega_* = \omega l^2\sqrt{\Omega/D}, \qquad p_* = \frac{\chi p_0 l^3}{D}\mathrm{M}, \qquad \varepsilon = \chi p_0 l^2/a_0\sqrt{\Omega D}, \qquad n_x = N_{xl}^2/D, \tag{1.8}$$

then (1.6) will be

$$a_6 = \vartheta k, \qquad a_4 = -(1+kn_x),$$

$$a_3 = kp_*,$$

$$a_2 = -k(\omega_*^2 - i\varepsilon\omega_*),$$

$$a_1 = -p_*, \qquad \lambda = \omega_*^2 - i\varepsilon\omega_*. \tag{1.9}$$

Represent the function $\chi(x)$ in (1.4) in the form

$$\chi(x) = e^{i\alpha x}, \tag{1.10}$$

where $\alpha$ is a root of the algebraic equation

$$a_6\alpha^6 - a_4\alpha^4 + ia_3\alpha^3 + a_2\alpha^2 - ia_1\alpha - \lambda = 0. \tag{1.11}$$

The general solution of (1.4), in the absence of multiple roots $\alpha_i$, has the form

$$\chi(x) = \sum_{i=1}^{6} c_i e^{i\alpha_i x}, \tag{1.12}$$

where $c_i$ are constants to be determined from the boundary conditions of the problem.

**§ 2.** Let us consider a number of boundary-value problems.

**A. Hinged edges.** In this case the conditions imposed on the displacement function $\chi$ at $x = 0,\ x = 1$ have the form (1, 2)

$$\chi = d^2\chi/dx^2 = d^4\chi/dx^4 = 0. \tag{2.1}$$

Substituting (1.12) into (2.1), we arrive at a system of homogeneous algebraic equations with respect to the coefficients $c_i$. The condition for a nontrivial solution will be

$$\Delta\delta^{-1} = 0, \tag{2.2}$$

where $\Delta(\alpha_1, \alpha_2, \dots, \alpha_6)$ is the determinant of the indicated system with respect to $c_i$; $\delta$ is the Vandermonde determinant formed from the roots $\alpha_i$.

Expanding (2.2), we obtain a sum of 10 terms of the form

$$\frac{(\alpha_6+\alpha_5)(\alpha_6+\alpha_4)(\alpha_5+\alpha_4)(\alpha_3+\alpha_1)(\alpha_2+\alpha_1)\sin(\alpha_1+\alpha_2+\alpha_3)}{(\alpha_6-\alpha_3)(\alpha_6-\alpha_2)(\alpha_6-\alpha_1)(\alpha_5-\alpha_3)(\alpha_5-\alpha_2)(\alpha_5-\alpha_1)(\alpha_4-\alpha_3)(\alpha_4-\alpha_2)(\alpha_4-\alpha_1)}. \tag{2.3}$$

Fig. 1. Variation of the real part of the reduced vibration frequency $\omega'_*$ of a three-layer rod (infinitely wide plate) in a supersonic gas flow; hinged edges; $\vartheta = 0.05;\ k = 1.00;\ V_* = 100.$

Figure 1: Fig. 1. Variation of the real part of the reduced vibration frequency $\omega'_*$ of a three-layer rod (infinitely wide plate) in a supersonic gas flow; hinged edges; $\vartheta = 0.05;\ k = 1.00;\ V_* = 100.$

Figure 2 graph

Figure 2: Figure 2 graph

The remaining terms of the sum can be obtained from (2.3) by permutation of the indices.

**Fig. 1.** Variation of the real part of the reduced vibration frequency $\omega'_*$ of a three-layer rod (infinitely wide plate) in a supersonic gas flow; hinged edges; $\vartheta = 0.05;\ k = 1.00;\ V_* = 100.$

The simultaneous solution of (1.11) and (2.3) makes it possible to find the complex frequency $\omega_*$ and the roots $\alpha_i$.

**B. Hinged support along the edges $x = 0,\ x = 1$ in the presence at the edges of diaphragms absolutely rigid in their own plane.** The boundary conditions for the displacement functions $\chi$ are written as $(^{1,2})$

$$\left(1 - k\frac{d^2}{dx^2}\right)\chi = \frac{d^2}{dx^2}\left(1 - \vartheta k\frac{d^2}{dx^2}\right)\chi = \frac{d^3\chi}{dx^3} = 0. \tag{2.4}$$

**Fig. 2.** Influence of the relative flexural rigidity of the face layers $\vartheta$ and of the shear parameter of the core $k$ on the real part of the reduced frequency of oscillations $\omega_*$ of a three-layer rod in a supersonic flow; one end of the rod is freely clamped, the other is hinged; $\vartheta = 0.01$ ($a$); 0.05 ($b$); 0.1 ($c$); $k = 1$–0.2; $v_* = 100$ (first two frequencies)

The left-hand side of (2.2) reduces to a sum of 10 terms of the form

$$\delta^{-1}[123][456]\left(e^{i(\alpha_1+\alpha_2+\alpha_3)} - e^{i(\alpha_4+\alpha_5+\alpha_6)}\right), \tag{2.5}$$

where

$$\begin{aligned} -[123]i = {} & \alpha_1^2\alpha_2^2(1 + k\alpha_3^2)(1 - \vartheta k\alpha_1\alpha_2)(\alpha_2 - \alpha_1) \\ & + \alpha_2^2\alpha_3^2(1 + k\alpha_1^2)(1 - \vartheta k\alpha_2\alpha_3)(\alpha_3 - \alpha_2) \\ & + \alpha_3^2\alpha_1^2(1 + k\alpha_2^2)(1 - \vartheta k\alpha_3\alpha_1)(\alpha_1 - \alpha_3), \end{aligned} \tag{2.6}$$

and an analogous expression for [456]. The remaining terms of the sum are obtained by the corresponding permutation of the indices.

**C. In the case of sagging hinged supports** the boundary conditions take the form

$$\frac{d^2\chi}{dx^2} = \frac{d^4\chi}{dx^4} = \left(1 - \vartheta k \frac{d^2}{dx^2}\right)\frac{d^3\chi}{dx^3} \pm c\chi = 0, \tag{2.7}$$

where $c$ is the stiffness coefficient of the support ($c = 0$—no supports; $c = \infty$—absolutely rigid support).

The condition for a nontrivial solution will be analogous to that given above (see (2.5)), while the factors in square brackets become the following:

$$\begin{aligned} -[123] = {} & \alpha_1^2\alpha_2^2(\alpha_2^2 - \alpha_1^2)\left[c - i\alpha_3^3(1 + \vartheta k\alpha_3^2)\right] \\ & + \alpha_2^2\alpha_3^2(\alpha_3^2 - \alpha_1^2)\left[c - i\alpha_1^3(1 + \vartheta k\alpha_1^2)\right] \\ & + \alpha_3^2\alpha_1^2(\alpha_1^2 - \alpha_3^2)\left[c - i\alpha_2^3(1 + \vartheta k\alpha_2^2)\right] \end{aligned} \tag{2.8}$$

and the corresponding expression for [456], with $c$ replaced by $-c$. The remaining terms are obtained from the first by permutation of the indices.

**D. Free clamping along the edge $x = 0$ and hinged support along the edge $x = 1$.** The boundary conditions are

$$\begin{aligned} x = 0: &\quad (1 - k\, d^2/dx^2)\,\chi = d\chi/dx = d^3\chi/dx^3 = 0; \\ x = 1: &\quad \chi = d^2\chi/dx^2 = d^4\chi/dx^4 = 0. \end{aligned} \tag{2.9}$$

The condition for a nontrivial solution is written in the form of equality to zero of a sum of 20 terms of the form

$$\delta^{-1}[456][123]e^{i(\alpha_1+\alpha_2+\alpha_3)}, \tag{2.10}$$

where

$$-[123] = \left[\alpha_1^4(\alpha_3^2 - \alpha_2^2) + \alpha_2^4(\alpha_1^2 - \alpha_3^2) + \alpha_3^4(\alpha_2^2 - \alpha_1^2)\right];$$

$$[456] = \alpha_4(\alpha_6^3 - \alpha_5^3) + \alpha_5(\alpha_4^3 - \alpha_6^3) + \tag{2.11}$$

$$+\, k\alpha_4\alpha_5\alpha_6\left[\alpha_4(\alpha_6^2 - \alpha_5^2) + \alpha_5(\alpha_4^2 - \alpha_6^2) + \alpha_6(\alpha_5^2 - \alpha_4^2)\right].$$

By the corresponding permutation of indices in (2.10), the remaining terms of the indicated sum can be obtained.

**Fig. 3.** Influence of the form of end fastening of a three-layer beam on the reduced frequency of oscillations $\omega_*$ in a supersonic flow; $\vartheta = 0.05$; $k = 1.00$; 1

Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

—one end is freely clamped, the other is free; 2 —one end is freely clamped, the other is free with a rigid diaphragm; 3 —hinged ends; 4 —hinged ends with rigid diaphragms; 5 —one end is freely clamped, the other is hinged; 6 —both ends are freely clamped

**Fig. 4.** Influence of the tension $n_x$ on the reduced frequency of oscillations of a three-layer beam in a supersonic flow; both ends hinged; $\vartheta = 0.05$; $k = 1.00$; $\nu_* = 100$ (the first two frequencies)

Similarly, an expression for $\Delta$ can also be obtained in the case of other forms of edge fastening.

Figures 1–4 present the results of calculations on the "Strela"electronic computer of the change in the reduced frequency of oscillations $\omega_*$ and the critical flutter velocity $p_*$ of three-layer two-dimensional panels (beams) in a supersonic flow as functions of various dimensionless parameters characterizing the three-layer panel and the gas flow passing over it.

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*Note: Figure translations are in progress. See original paper for figures.*