AN ITERATIVE PRINCIPLE FOR CONSTRUCTING MULTICHANNEL AUTOMATIC CONTROL SYSTEMS

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Abstract

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1°. To improve the quality of operation of automatic control systems, various multichannel and multidimensional systems are widely used, in which, in one form or another, additional information is employed about the useful signal, disturbances, or the dynamic characteristics of individual elements ([1−3]). Let us consider the principle of construction and the dynamic equations of a multichannel system intended for the precise reproduction of a prescribed input $x(t)$, in which the output of each of the channels would be used as a correction to refine the value of the useful signal measured by means of all preceding channels.

Let the multichannel system contain n channels described by linear differential equations

$$C_k(D)y_k(t) = B_k(D)x_k(t), \qquad C_k(D)\delta_k(t) = A_k(D)x_k(t),$$
$$k = 1, 2, \dots, n, \tag{1}$$

where $x_k(t),\ y_k(t)$ are the input and output quantities of the channels; $\delta_k(t) = x_k(t) - y_k(t)$ are the reproduction errors; $A_k(D),\ B_k(D),\ C_k(D)$ are operational polynomials; $C_k(D) = A_k(D) + B_k(D)$.

If $y_{N-1}(t)$ is the output of a system consisting of $(n-1)$ channels, then the output $y_N(t)$ of a system containing one more channel $(N = n)$ is taken to be

$$y_N(t) = y_{N-1}(t) + y_n(t), \tag{2}$$

where $y_n(t)$ is the output quantity of the n-th channel (1), $k = n$.

In order that the n-th channel should actually introduce a correction $y_n(t)$ making it possible to reproduce the prescribed input $x(t)$ more accurately, we take as its equivalent input the error $\varepsilon_{N-1}(t)$ of the $(n-1)$-channel system

$$x_n(t) = \varepsilon_{N-1}(t) = x(t) - y_{N-1}(t). \tag{3}$$

Multichannel systems that solve the problem of reproducing the useful signal by successive approximations carried out by the corresponding channels will be called iterative. Since the operating algorithm is specified by relations (1)–(3), their fundamental equations can be determined for the output quantity $y_N(t)$ and the error $\varepsilon_N(t) = x(t) - y_N(t)$:

$$C_N(D) y_N(t) = B_N(D) x(t), \qquad C_N(D) \varepsilon_N(t) = A_N(D) x(t), \tag{4}$$

where

$$C_N(D) = C_{N-1}(D) C_n(D), \qquad A_N(D) = A_{N-1}(D) A_n(D), \tag{5}$$

$$B_N(D) = B_{N-1}(D) C_n(D) + C_{N-1}(D) B_n(D) - B_{N-1}(D) B_n(D). \tag{6}$$

Thus, in an iterative multichannel system there is no mutual influence of the channels from the point of view of stability, and its order of astatism v_N is equal to the sum of the orders of astatism $v_1, v_2, v_3, v_4, \dots, v_n$ of the individual

channels, which to a considerable extent determines the possibility of achieving high accuracy in reproducing $x(t)$.

2°. Let us take into account the possibility of disturbances $f_k(t)$ arising in successive approximations; in this connection, as the equivalent input quantity $x_n(t)$ in (3) we shall take

$$x_n(t) = x_n^*(t) = x(t) - y_{N-1}(t) + f_n(t). \tag{7}$$

Denoting the channel operators (1) by

$$W_k(D) = B_k(D)/C_k(D), \qquad E_k(D) = A_k(D)/C_k(D),$$

$$R_k(D) = B_k(D)/A_k(D), \tag{8}$$

from (1), (2), (4)–(8) we find

$$\varepsilon_N(t) = \prod_{k=1}^{n} E_k(D) x(t) - \sum_{k=1}^{n} \prod_{i=k+1}^{n} E_i(D) W_k(D) f_k(t), \qquad n \geqslant 2. \tag{9}$$

In the case of iterative systems with noise accumulation, when the equivalent noise at the k-th iteration stage contains all preceding noises

Fig. 1

Figure 1: Fig. 1

Fig. 1

$\varphi_1(t), \varphi_2(t), \dots, \varphi_{k-1}(t)$ plus its own noise $\varphi_k(t)$, we obtain

$$\varepsilon_N(t) = \prod_{k=1}^{n} E_k(D)x(t) - \sum_{k=1}^{n}\left[1 - \prod_{i=k}^{n} E_k(D)\right]\varphi_k(t), \qquad n \geqslant 2. \qquad (10)$$

Relations (9), (10) are the starting point for the analysis and synthesis of iterative systems with allowance for disturbances.

3°. A very great variety of structural diagrams of iterative systems is possible (for example, Fig. 1); in this case the methods of connecting the individual channels must satisfy equalities (5), (6). All types of iterative systems, as follows from (9), (10), are equivalent from the point of view of compensating the influence of individual channels on the errors with respect to the prescribed input action and allow, by the choice of the operators $W_k(D)$ (8), high accuracy of reproduction of $x(t)$ to be ensured.

In particular, if each of the channels has first-order astatism ($v_k = 1$) and gain factor 100, then the steady-state dynamic error $\varepsilon_g(t)$ in reproducing the useful signal $x(t)$, represented by a polynomial of the third degree, in a three-channel iterative system without disturbances, according to (9) or (10), is equal to $\varepsilon_g(t) = 10^{-6}\, d^3x/dt^3$. To achieve an error just as small, an equivalent single-channel closed-loop system would have to possess third-order astatism and a gain factor of 10^6, which is practically unrealizable.

It follows from (9), (10) that, whereas in iterative systems without accumulation of the noise $f_k(t)$ (Fig. 1) the corresponding component of the error (9) decreases (is compensated) as the number of channels grows, in systems with accumulation of noise the error component due to the noise $\varphi_k(t)$ (10) increases.

In ([4],[5]) structural schemes, dynamic and random errors of a two-channel system are considered, as well as self-adjustment of the channel parameters with respect to the input signal. Iterative systems can be effectively applied in measuring devices when there is redundant information about the useful signal, and also in control systems for moving objects ([2],[5]).

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Note: Figure translations are in progress. See original paper for figures.