QUASIOPTICAL LINES WITH BOUNDED UNEQUALLY SPACED CORRECTORS

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QUASIOPTICAL LINES WITH BOUNDED UNEQUALLY SPACED CORRECTORS

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1. In paper (1), quasioptical transmission lines were proposed consisting of identical phase correctors located at equal distances from one another. In such beam guides with equidistant correctors, which transform fields according to*

$$u^+(x) = \begin{cases} u^-(x)\exp[ikx^2/2f], & |x| \leqslant a, \\ 0, & |x| > a, \end{cases}$$

there exists a system of eigenwaves. (Here u^- (u^+) is the field at the input (output) of the corrector; k is the wave number; f is the focal length of the corrector, and a is the half-width of its aperture.) The fields of the eigenwaves are introduced in the mean planes of the phase correctors:

$$u(x) = u^-(x)\exp[ikx^2/2(2f)] = u^+(x)\exp[-ikx^2/2(2f)] \tag{1}$$

and satisfy the integral equation:

$$\chi_n\varphi_n(\eta) = \sqrt{\frac{C}{2\pi}}\int_{-1}^{1}\varphi_n(y)\exp\{iC[y\eta - \tfrac{1}{2}g(y^2+\eta^2)]\}\,dy, \tag{2}$$

where $u(x) = a^{-1/2}\varphi(x/a)$; $C = ka^2/L$; L is the distance between the correctors of the line and $g = 1 - L/2f$. The diffraction losses and the phase increment, determined by the modulus and phase of the number χ_n, as well as the field of the n-th eigenwave $u_n(x) = a^{-1/2}\varphi_n(C;\, g;\, x/a)$, $|x| \leq a$, depend only on the two parameters of the line C and g and are unchanged for all iterations.

The distances between the correctors of a real transmission line are dictated by the terrain and, generally speaking, it is convenient to place phase correctors at unequal distances from one another. A beam guide with unequally spaced correctors has no eigenwaves, i.e., there are no field structures that repeat at each of the correctors of the beam guide.

Let us broaden the concept of an eigenwave. We shall call a wave beam a generalized eigenwave of a quasioptical line if the fields of this beam at any two correctors of the line are obtained from one another by a similarity transformation, and if at each iteration the magnitude of its phase increment is constant. The diffraction losses of this wave beam per iteration also turn out to be constant. As will be shown below, for any distances between correctors it is possible to choose their focal lengths and aperture sizes so that the unequally spaced beam guide will have generalized eigenwaves.

2. In this section, relations are obtained that express the focal lengths and aperture sizes of the correctors of an unequally spaced beam guide having generalized eigenwaves through the prescribed distances l_j between the correctors.

* For simplicity, a two-dimensional problem is considered. The results are readily generalized to the three-dimensional case.

The field $W^j(x)$, existing in the median plane of the j-th corrector of the line, produces, according to Huygens' principle, in the median plane of the $(j+1)$-st corrector the field

$$W^{j+1}(\xi) = \exp[i\pi/4]\frac{\exp[-\frac{1}{2}ikl_{j+1}]}{\sqrt{\lambda l_{j+1}}}\times$$

$$\times\int_{-a_j}^{a_j} W^j(x)\exp\left[-\frac{ik}{2l_{j+1}}(x-\xi)^2\right]\exp\left[-\frac{ik}{2f_j^{(2)}}x^2\right]\exp\left[-\frac{ik}{2f_{j+1}^{(1)}}\xi^2\right]dx, \quad (3)$$

where a_j is the half-width of the aperture of the j-th corrector, and l_{j+1} is the distance between the j-th and $(j+1)$-st correctors. Choosing the focal distances $f_j^{(2)}$ of the second "half" * of the j-th corrector and $f_{j+1}^{(1)}$ of the first "half" of the $(j+1)$-st corrector so that the kernel of the integral transformation (3) is the same for all iterations $(j = 0, 1, 2, ...)$, and requiring similarity of the fields at two neighboring correctors,**

$$\left(\frac{a_{j+1}}{a_j}\right)^{1/2} W^{j+1}(a_{j+1}\eta) = \chi W^j(a_j\eta), \quad |\eta| \le 1,$$

we obtain the integral equation for the generalized eigenwaves

$$\chi_n W_n^j(a_j\eta) = \sqrt{\frac{C}{2\pi}}\int_{-1}^{1} W_n^j(a_{jy})\exp\{iC[y\eta - \tfrac{1}{2}g(y^2+\eta^2)]\}\,dy, \quad (4)$$

where $f_j^{(2)}$, $f_{j+1}^{(1)}$, and a_j must be equal to

$$1/f_j^{(2)} = 1/l_{j+1} - g/L_j, \quad 1/f_{j+1}^{(1)} = 1/l_{j+1} - g/L_{j+1}; \tag{5}$$

$$a_{j+1}/a_j = l_{j+1}/L_j; \tag{5'}$$

the quantity L_j is introduced by the iterative relation

$$L_j = l_j^2/L_{j-1},$$

and C in (4) is equal to

$$C = ka_{ja_{j+1}}/l_{j+1} = ka_j^2/L_j.$$

Here the fundamental parameters are C, g, and k, which are fixed for the entire nonequidistant beam guide. The quantity L_j in relations (5) is equal to the distance between the correctors of a certain equidistant beam guide whose eigenwave fields are determined by the functions $W_n^j(x)$.

The fields of the generalized eigenwaves, as is seen from comparing the integral equations (4) and (2), coincide with the fields of the ordinary eigenwaves

$$W_n^{j+1}(\xi) = (a_{j+1})^{-1/2}\varphi_n(C; g; \xi/a_{j+1}), \quad |\xi| \leq a_{j+1},$$

differing from the latter only by a change of scale, i.e., of the quantities a_{j+1}, from iteration to iteration.

From relations (5) it is easy to obtain the total focal distances of the correctors of a quasioptical line having generalized eigenwaves:

$$1/F_j = 1/f_j^{(1)} + 1/f_j^{(2)} = 1/l_j + 1/l_{j+1} - 2g/L_j. \tag{6}$$

As is seen from formulas (5) and (6), the focal distances and aperture sizes of the correctors depend (for $g \neq 0$) also on the excitation conditions of the beam –

* The quotation marks are used because the field in the median plane of the j-th corrector of a nonequidistan

$$W(x) = W^-(x)\exp\left[ik\frac{x^2}{2f_j^{(1)}}\right] = W^+(x)\exp\left[-ik\frac{x^2}{2f_j^{(2)}}\right]$$

(cf. (1)) and $f_j^{(1)} \neq f_j^{(2)}$, if $l_j \neq l_{j+1}$ and $g \neq 0$.

** The normalizing factor before W^{j+1} is introduced so that χ has the same simple physical meaning as in an equidistant beam guide.

parameter specified by the quantity L_0 (or a_0), which determines the scale of the fields of the generalized eigenwaves at the beginning of the line.

3. Generalized eigenwaves exist, in particular, in equidistant beam waveguides whose parameters are determined by formulas (5) and (6) for $l_j \equiv l$, and in beam waveguides with unlimited apertures. It is well known ([2],[3]) that a Gaussian beam of any order, when propagating in any quasioptical lines with unlimited quadratic correctors, remains a Gaussian beam. However, only a beam waveguide whose corrector focal distances are equal to (6) has generalized eigenwaves. These waves differ from all other Gaussian beams in that the transformation of the spot sizes of the wave fields from corrector to corrector is given (also for $a_j = \infty$, $a_{j+1} = \infty$) by the right-hand side of (5′), while the value of the phase advance of each wave is constant over all iterations and is determined by the phase of the number χ_n.

Consideration of an irregular transmission line with phase correctors randomly shifted in the transverse direction shows that in a non-equidistant beam waveguide it is possible to damp the growth of the displacement of the center of a quasioptical beam by a corresponding arrangement of the correctors.

In conclusion, we note that non-equidistant beam waveguides of the class considered can be used not only as transmission lines. They make it possible, for example, to solve the problem of matching two equidistant quasioptical lines in which the distances between correctors are different, or the problem of exciting a transmission line by means of a laser. (The corresponding formulas are readily obtained from (5) and (6).) The problem in this case consists in using completely definite fields–the fields of eigenwaves–to excite exactly the same fields, changing only their scale–in contrast to the formulation of the problem in ([4]), where it is required, by means of an arbitrary field, to excite the desired field as accurately as possible.

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Note: Figure translations are in progress. See original paper for figures.