Aerodynamics

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Abstract

Full Text

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HIGH-RANK MOMENTS IN A WEAKLY RAREFIED GAS

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When a gas leaves the state of local equilibrium, the moments of the distribution function change, generally speaking, for all ranks ([1]). In the present work a 56-moment system of equations ([2]) is solved for small Knudsen numbers with an arbitrary interaction potential. All elements of the moment tensors of the second, third, fourth, and fifth ranks are expressed in terms of the principal moments: density, velocity, and temperature. Thus the internal structure of these tensors is revealed and the relations between them in a weakly rarefied gas are found.

1. Let the distribution function $f(\mathbf{r}, \mathbf{u}, t)$ be expanded in a series in the Hermite polynomials $H^{(m)}(\mathbf{v})$

$$f(\mathbf{r}, \mathbf{u}, t) = f_0(\mathbf{r}, \mathbf{u}, t) \sum_{m=0}^{\infty} \frac{1}{m!} \sum_{i_1,\ldots,i_m=1}^{3} a^{(m)}_{i_1,\ldots,i_m}(\mathbf{r}, t) H^{(m)}_{i_1,\ldots,i_m}(\mathbf{v})$$

with weight function

$$f_0(\mathbf{r}, \mathbf{u}, t) = n(\mathbf{r}, t)[h(\mathbf{r}, t)/\pi]^{3/2} \exp(-v^2/2),$$

where $\mathbf{v} = \sqrt{2h(\mathbf{r}, t)}[\mathbf{u} - \mathbf{U}(\mathbf{r}, t)]$.

By virtue of the orthogonality of the Hermite polynomials,

$$a^{(m)}_{i_1,\ldots,i_m}(\mathbf{r}, t) = \frac{1}{n(\mathbf{r}, t)} \iiint_{-\infty}^{+\infty} f(\mathbf{r}, \mathbf{u}, t) H^{(m)}_{i_1,\ldots,i_m}(\mathbf{v})\, d\mathbf{u},$$

where $a^{(0)} = 1, \quad a^{(1)}_i = 0, \quad a^{(2)}_{ii} = 0.$

For small Knudsen numbers the coefficients $a^{(m)}$, $m > 1$, are small. To first-order accuracy in small quantities, the system of equations from ([2]) for $a^{(m)}$, $m = 2, 3, 4, 5$, can be written in the form

$$\frac{\delta_{ij}}{h}\frac{Dh}{Dt}-\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)=-\frac{16}{5\sqrt{\pi}}\,\frac{n\sigma_0 b_{11}}{\sqrt{2h}}\left[a_{ij}+B_1\left(a_{ijkk}-\frac{\delta_{ij}}{3}a_{kkll}\right)\right]; \quad (1)$$

$$\frac{2}{(2h)^{3/2}}\,3\{\delta_i\delta_{kl}\}_s\frac{\partial h}{\partial x_l}=-\frac{24}{5\sqrt{\pi}}\,\frac{n\sigma_0 b_{11}}{\sqrt{2h}}\left[a_{ijk}-\frac{1}{9}\cdot 3\{\delta_{ij}a_{kll}\}_s+B_1\left(a_{ijkll}-\frac{1}{9}\cdot 3\{\delta_i a_{kllmm}\}_s\right)\right]; \quad (2)$$

$$0=2B_1\,6\{\delta_{ij}a_{kl}\}_s+\left(\frac{3}{2}+\frac{20}{27}B_3\right)a_{ijkl}+ \quad (3)$$

$$+\left(-\frac{1}{12}+\frac{B_2}{2}-\frac{20}{189}B_3\right)6\{\delta_{ij}a_{klmm}\}_s+\left(-\frac{B_2}{3}+\frac{4}{189}B_3\right)3\{\delta_{ij}\delta_{kl}a_{mmnn}\}_s;$$

$$0=2B_1\left(10\{\delta_{ij}a_{klm}\}_s-\frac{2}{9}\cdot 15\{\delta_{ij}\delta_{kl}a_{mnn}\}_s\right)+\left(\frac{5}{6}+\frac{100}{81}B_3\right)a_{ijklm}+ \quad (4)$$

$$+\left(-\frac{1}{36}+\frac{B_2}{2}-\frac{20}{189}B_3\right)10\{\delta_{ij}a_{klmnn}\}_s+\left(-\frac{B_2}{9}+\frac{4}{527}B_3\right)15\{\delta_{ij}\delta_{kl}a_{mnnpp}\}_s.$$

Here $\{...\}_s$ is the symmetrization symbol;

$$B_1=\frac{2b_{12}}{7b_{11}}-\frac{1}{4},\qquad B_2=\frac{20}{49}\frac{b_{13}}{b_{11}}-\frac{4b_{12}}{7b_{11}}+\frac{1}{4},\qquad B_3=\frac{10}{7}\frac{b_{13}}{b_{11}}+\frac{b_{23}}{b_{11}};$$

$$b_{kl}=\frac{4\pi(2k+1)!!}{(l+2)!(2k)!!}\int_0^\infty\int_0^{\rho_{\max}} w^{2l+5}e^{-w^2}\sin^{2k}\chi\,\frac{\rho\,d\rho}{\sigma_0}\,dw$$

are collision functionals between two atoms with velocities $\mathbf{u}_1$ and $\mathbf{u}_2$; ρ is the impact parameter; $w=\sqrt{h}/2(\mathbf{u}_2-\mathbf{u}_1)$; $\chi=\chi(\rho,|\mathbf{u}_2-\mathbf{u}_1|)$ is the angle of deflection of w as a result of the collision, determined by the interaction potential; $\sigma_0=\sigma_0(h)$ is the characteristic collision cross section. If the potential is not finite but decreases at infinity faster than the Coulomb potential, then as $\rho_{\max}\to\infty$, $\sigma_0 b_{11}$ remains finite, and $\sigma_0 b_{kl}$, $k>1$, all the more so.

2. Solving equations (1)–(4) with respect to $a^{(m)}$, $m=2,3,4,5$, we obtain

$$a_{ij} = \frac{5\sqrt{\pi}}{16} \frac{\sqrt{2h}}{n\sigma_0 b_{11}} \frac{1}{(1-7B_1C_1)} \left[\frac{\delta_{ij}}{h} \frac{Dh}{Dt} - \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right];$$

$$\frac{1}{3} a_{111} = a_{122} = a_{133} = \frac{15\sqrt{\pi}}{32} \frac{1}{n\sigma_0 b_{11}} \frac{1}{(1-7B_1C_2)} \frac{1}{h} \frac{\partial h}{\partial x_1},$$

$$\frac{1}{3} a_{222} = a_{112} = a_{233} = \frac{15\sqrt{\pi}}{32} \frac{1}{n\sigma_0 b_{11}} \frac{1}{(1-7B_1C_2)} \frac{1}{h} \frac{\partial h}{\partial x_2},$$

$$\frac{1}{3} a_{333} = a_{113} = a_{223} = \frac{15\sqrt{\pi}}{32} \frac{1}{n\sigma_0 b_{11}} \frac{1}{(1-7B_1C_2)} \frac{1}{h} \frac{\partial h}{\partial x_3},$$

$$a_{123} = 0;$$

$$^1/_6 a_{1111} = -a_{2233} = -C_1 a_{11}, \qquad a_{2223} = a_{2333} = 3a_{1123} = -3C_1 a_{23},$$

$$^1/_6 a_{2222} = -a_{1133} = -C_1 a_{22}, \qquad a_{1113} = a_{1333} = 3a_{1223} = -3C_1 a_{13},$$

$$^1/_6 a_{3333} = -a_{1122} = -C_1 a_{33}, \qquad a_{1112} = a_{1222} = 3a_{1233} = -3C_1 a_{12};$$

$$^1/_5 a_{11111} = a_{12222} = a_{13333} = a_{11122} = a_{11133} = 3a_{12233} = -3C_2 a_{122},$$

$$^1/_5 a_{22222} = a_{11112} = a_{23333} = a_{11222} = a_{22233} = 3a_{11233} = -3C_2 a_{112},$$

$$^1/_5 a_{33333} = a_{11113} = a_{22223} = a_{11333} = a_{22333} = 3a_{11223} = -3C_2 a_{113},$$

$$a_{11123} = a_{12223} = a_{12333} = 0,$$

where

$$C_1 = \frac{12(-7b_{11} + 8b_{12})}{301b_{11} - 336b_{12} + 240b_{13}}, \qquad C_2 = \frac{4(-7b_{11} + 8b_{12})}{77b_{11} - 112b_{12} + 80b_{13}}.$$

It is worth noting that

$$a_{ijjj} = 0, \qquad a_{ijhh} = -7C_1 a_{ij}, \qquad a_{ijjhk} = -7C_2 a_{ij}.$$

For spherical atoms $b_{kl} = 1$, $B_1 = {}^1/_{28}$, $B_2 = {}^{17}/_{196}$, $B_3 = {}^3/_7$, $C_1 = {}^{12}/_{205}$, $C_2 = {}^4/_{45}$, and the numerical values of the coefficients $(1 - 7B_1C_1)^{-1} = 1{}^3/_{202}$, $(1 - 7B_1C_2)^{-1} = 1{}^1/_{44}$ correspond to the second approximation of the Enskog–Chapman method for the coefficients of viscosity and thermal conductivity (3).

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named after A. A. Zhdanov

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2. R. G. Barantsev, *Aerodynamics of Rarefied Gases*, Collection II, Leningrad, 1965, p. 98.
3. S. Chapman, T. Cowling, *The Mathematical Theory of Non-Uniform Gases*, Moscow, 1960, p. 205.

Note: Figure translations are in progress. See original paper for figures.