THE VELOCITY FIELD EXCITED BY A SHOCK WAVE IMPINGING ON A STATIONARY THIN SYMMETRIC BODY

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Fig. 1

Figure 1: Fig. 1

Abstract

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THE VELOCITY FIELD EXCITED BY A SHOCK WAVE IMPINGING ON A STATIONARY THIN SYMMETRIC BODY

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1. The velocity field excited by a shock wave of finite amplitude impinging on a stationary obstacle is investigated (^{1}).

We shall assume that a shock wave of finite amplitude impinges on a stationary thin body located inside an unbounded volume of a compressible medium. Let the body be symmetric with respect to a plane. The projection Σ of the body onto the plane of symmetry is bounded by a curve *AOBD* of arbitrary form (Fig. 1). We shall assume that the front of the shock wave is a plane moving perpendicular to the plane of symmetry of the body with velocity $v > a_0$, where a_0 is the speed of sound in the undisturbed gas.

Fig. 1

Let us take a fixed system of coordinate axes, as indicated in Fig. 1. We place the plane xOy in the plane of symmetry of the body. We direct the axis Ox opposite to the direction of motion of the wave front. Considering the perturbations of the medium excited by the presence of the body to be small, we consider the problem of determining the velocity field behind the front of the shock wave in a linearized formulation (^{2,3}).

We shall assume the gas motion to be irrotational. The velocity potential Φ satisfies the wave equation

$$\Phi_{xx} + \Phi_{yy} + \Phi_{zz} - \frac{1}{a^2}\Phi_{tt} = 0, \tag{1}$$

where a is the speed of sound behind the shock-wave front.

On the surface of the body the flow condition is satisfied,

Fig. 2

Figure 2: Fig. 2

$$\Phi_n = v_{0n} = 0. \tag{2}$$

We transfer condition (2) parallel to the axis Oz onto the plane xOy. We shall seek the velocity potential Φ in the form

$$\Phi(x, y, z, t) = \varphi_w(x, t) + \varphi(x, y, z, t). \tag{3}$$

The function φ_w satisfies equation (1) and is a prescribed function. By assumption, the function φ_w differs little from the function determining the velocity potential of a translational flow with velocity u, i.e., $\varphi_{wx} = u$ to within small quantities of first order. The desired potential φ is a solution of equation (1) satisfying the boundary conditions in the plane xOy.

In the region Σ behind the shock-wave front,

$$\varphi_z = -\varphi_{wx}(x, t)\beta(x, y) = A_w(x, y, t), \tag{4}$$

the function β—the angle of attack of the elements of the upper surface of the body—is small and is an arbitrary continuous function of its arguments.

Everywhere in the plane xOy outside the region Σ, and also inside Σ ahead of the wave front, the condition is satisfied

$$\varphi_z(x, y, 0, t) = 0. \tag{5}$$

It is sufficient to solve the problem for the upper half-space ($z > 0$). The solution of the problem in the lower half-space is found from the condition $\varphi(x, y, -z, t) = \varphi(x, y, z, t)$, since the function φ is even.

2. To solve the problem we shall apply the method proposed in papers ([4],[5]). Consider the three-dimensional region V in the space xyt (Fig. 2). The region V

Fig. 2

is bounded by the surface Σ^*. In the case of a stationary body the surface Σ^* is a cylindrical surface with generators parallel to the axis Ot. The directrix of Σ^* is the contour $AOBD$. Let the contour $AOBD$ be given by the equation

$$\eta = \Psi(\xi), \tag{6}$$

Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

where Ψ is an arbitrary continuous function. Equation (6) is at the same time the equation of the surface Σ^* in the space xyt. Inside and on the boundary of the region V, the derivative φ_z is prescribed according to the flow-tangency condition.

Let the front of the shock wave move according to the law $x = f(t)$, where f is a prescribed function, with $v = f'(t)$. The surface W, defined by the equation $\xi = f(\tau)$, divides the region V into two parts with different values of the derivative φ_z. To the left of the surface W the derivative $\varphi_z = 0$, and to the right $\varphi_z = A_w$ (Fig. 2).

We take the solution of the wave equation in the form (^{4})

$$\varphi(x,y,z,t) = -\frac{a}{2\pi} \iint\limits_{S(x,y,z,t)} \frac{\varphi_z(\xi,\eta,0,\tau)}{R}\, dS, \tag{7}$$

$$\tau = t - a^{-1}\sqrt{(x-\xi)^2 + (y-\eta)^2 + z^2}, \quad R = \sqrt{(1+a^2)[(x-\xi)^2 + (y-\eta)^2 + a^2 z^2]}.$$

The surface S is a hyperboloid defined by the equation

$$(x-\xi)^2 + (y-\eta)^2 + z^2 - a^2(t-\tau)^2 = 0 \tag{8}$$

and by the inequality $\tau < t$.

We represent the solution (7) in the form

$$\varphi(x,y,z,t) = -\frac{a}{2\pi} \iint\limits_{S^*(x,y,z,t)} \frac{A_w(\xi,\eta,\tau)}{R}\, dS, \tag{9}$$

where the region of integration S^* is the part of the surface S that belongs to the region V and is situated to the right of the surface W (Fig. 2).

Thus, formula (9) gives the solution of the posed problem in closed form, in the form of a surface integral.

Fig. 3

Fig. 4

In formula (9) we pass from the surface integral to a double integral with a plane region of integration in the plane xOy, using the relation $dS = \sqrt{EG - F^2}\, d\xi\, d\eta$, where the quantities E, G, F are the coefficients of the differential elements in the first fundamental quadratic form. Then we obtain

$$\varphi(x, y, z, t) = \frac{1}{2\pi} \iint\limits_{\sigma} \frac{A_w\Big(\xi, \eta, t - a^{-1}\sqrt{(x-\xi)^2 + (y-\eta)^2 + z^2}\Big)}{\sqrt{(x-\xi)^2 + (y-\eta)^2 + z^2}}\, d\xi\, d\eta. \quad (10)$$

The region of integration σ is the part of the region Σ situated inside the curve l. The curve l is the projection onto the plane xOy of the line of intersection of the surfaces S and W. The curve l divides the region Σ into parts with different values of the derivative φ_z. If the front of the shock wave moves with constant supersonic velocity $v =$ const, then the surface W is a plane defined by the equation $\xi + v\tau = 0$, and the curve l is an ellipse (Fig. 3). The equation of the ellipse has the form

$$(v^2 - a^2)\xi^2 + v^2\eta^2 - 2v(vx + a^2t)\xi - 2v^2y\eta + v^2(x^2 + y^2 + z^2 - a^2t^2) = 0. \quad (11)$$

3. Let us refer to the instant of time t. In the space $xy\tau$ draw the plane $\tau = t$ (Fig. 2). The projection of the line of intersection of the plane $\tau = t$ with the plane W onto the plane xOy will also be denoted by the letter W (Fig. 4).

Consider in the space $xy\tau$ the family of cones defined by equation (8) for $z = 0$ and by the inequality $\tau > t$, with vertices on the line of intersection of the surfaces W and Σ^*. Denote the envelope surface of this family by Ω (Fig. 2). The projections of the lines of intersection of the plane $\tau = t$ with the envelope Ω onto the plane xOy will be denoted by Ω_1 and Ω_2. In the case corresponding to $v =$ const, we find the equations of the curves Ω_1 and Ω_2 in para-

in parametric form

$$v^2(x^* - \xi)^2 + v^2[\Psi(x^*) - \eta]^2 - a^2(x^* + vt)^2 = 0,$$

$$v^2(x^* - \xi) + v^2[\Psi(x^*) - \eta]\Psi'(x^*) - a^2(x^* + vt) = 0, \quad (12)$$

where x^* is the parameter. Of the two real solutions of equations (11), the curve Ω_1 corresponds to the solution with the smaller value of the variable ξ, and the curve Ω_2 to that with the larger value of this variable.

The lines W, Ω_1, Ω_2 divide the regions with different analytic character of the solution of the problem (Fig. 4). In the region bounded by the straight line

W and the curve Ω_1, in solution (10) the domain of integration σ is bounded by the ellipse (11), located inside Σ (Fig. 3). With time this region disappears. In the region bounded by the curves Ω_1 and Ω_2, in solution (10) the domain σ is the part of Σ cut off by the ellipse. In this case the ellipse intersects the contour $AOBD$ (Fig. 3). With time there appears a region in which the coordinates of the points of intersection of the ellipse with the contour $AOBD$ become imaginary. The region Σ is then entirely located inside the ellipse (11). This region corresponds to solution (10), where the domain of integration is the entire region Σ, bounded by the contour $AOBD$, i.e., the region σ coincides with Σ.

Outside the region indicated by hatching in Fig. 4, at the time t, the influence of the shock wave is not felt.

We note that for $v = a$ the curve l is a parabola with axis parallel to the direction of motion of the wave front.

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Note: Figure translations are in progress. See original paper for figures.