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Fig. 1

Figure 1: Fig. 1

Abstract

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FRESNEL DIFFRACTION WITH A NON-SPHERICAL WAVE

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Fresnel diffraction from opaque objects with a rectilinear edge (a narrow screen, a slit, a half-plane) is considered for the case when the wave surface is not a sphere but a paraboloid or a parabolic cylinder. Under certain conditions, in this case the phenomenon of transition of the Fresnel diffraction pattern into the geometrical shadow is observed.

1. Let a light wave W be incident on a narrow opaque screen of width d (Fig. 1). In free propagation of the wave the amplitude in the plane xOy, passing through the surface of the screen, is determined by the relation

Fig. 1

$$A(x,y) = \exp\left[i\frac{2\pi}{\lambda}P(x,y)\right], \tag{1}$$

where $P(x,y) = \frac{1}{2}P_{xx}x^2 + \frac{1}{2}P_{yy}y^2$ describes the phase distribution. In that part of the plane xOy where the derivatives $P(x,y)$ are small, the equation $z = P(x,y)$ may be taken as the equation of the wave surface. The quantities $h_x = 1/P_{xx}$ and $h_y = 1/P_{yy}$ are the applicates of the centers of curvature of the principal normal sections at the vertex of the wave.

Let us determine the light intensity in the plane $z = h$, assuming here that $h \neq h_x$, $h \neq h_y$. Using the Huygens-Fresnel principle, we find [1] that, depending on the position of the observation plane, there are two different distributions of light intensity.

a) The observation plane R does not pass through O_ψ, the center of curvature of the normal section C_ψ, parallel to the edge of the screen and making an angle ψ with the plane of symmetry of the wave (Fig. 1). Then

$$1/h \neq \cos^2\psi/h_x + \sin^2\psi/h_y,$$

and in the plane R a Fresnel diffraction pattern is formed with intensity

$$I(l) = \frac{1}{2}\frac{|h_x||h_y|}{|h-h_x||h-h_y|}\left[\left(1-\int_{vl-\mu}^{vl+\mu}\sin\frac{\pi}{2}t^2\,dt\right)^2 + \left(1-\int_{vl-\mu}^{vl+\mu}\cos\frac{\pi}{2}t^2\,dt\right)^2\right], \tag{2}$$

where

$$v = \left[\frac{2}{\lambda}\,\frac{|h_x|^2|h-h_y|^2\sin^2\psi + |h_y|^2|h-h_x|^2\cos^2\psi}{h|h_x||h_y||h-h_x||h-h_y|\{1-h(\cos^2\psi/h_x+\sin^2\psi/h_y)\}}\right]^{1/2}, \tag{3}$$

$$\mu = d\left[\frac{1}{2\lambda}\,\frac{|h-h_x||h-h_y|}{h|h_x||h_y||1-h(\cos^2\psi/h_x+\sin^2\psi/h_y)|}\right]^{1/2}, \tag{4}$$

l is the distance from the observation point to the axis of symmetry of the diffraction pattern—a straight line with angular coefficient

$$k = \operatorname{tg}\psi\,[h_x(h-h_y)/h_y(h-h_x)].$$

The quantity μ is a parameter of the distribution of light intensity. Diffraction patterns with a fixed value of μ are geometrically similar. The width of the diffraction patterns is inversely proportional to v.

b) The observation plane R_ψ passes through O_ψ. In this case

$$1/h = \cos^2\psi/h_x + \sin^2\psi/h_y,$$

and the Fresnel diffraction pattern passes into a geometrical shadow: outside the shadow the light intensity is equal to

$$I_\Gamma = |h_x||h_y|/|h-h_x||h-h_y|$$

(the intensity under free propagation of the wave), while inside the shadow it is zero. The angle between the screen and the shadow is a straight one.

Let us note that the conclusion about the transition of the Fresnel diffraction pattern into a geometrical shadow follows directly from (3) and (4). Indeed, as

$$1/h \to \cos^2 \psi / h_x + \sin^2 \psi / h_y$$

the denominator of the radical expressions in these formulas tends to zero, which also occurs as $\lambda \to 0$, i.e., in the transition to geometrical optics.

Replacing the narrow screen by a slit or by a half-plane does not change the result. These objects also give a geometrical shadow situated perpendicular to their edge.

2. The transition of the Fresnel diffraction pattern into a geometrical shadow may be explained as follows. The light intensity at each point of the observation plane is the result of the superposition of secondary elementary Fresnel waves emitted by portions of the wave front not covered by the opaque object.

The result can be determined in the following sequence. First, the secondary Fresnel waves emitted by elements of the wave front situated on an infinite straight line H, parallel to the edge of the object D (Fig. 2), are summed. The second summation consists in adding the light disturbance produced by infinite straight lines lying outside the object.

It can be shown that the total action of the secondary Fresnel waves from the elements of the infinite straight line H may be replaced by the action of a specially chosen wave from some point M on this straight line.

The point M is found by a simple construction. Mark in the plane xOy a point B, which geometrical optics brings into correspondence with the chosen

point in the observation plane. Through the point B we draw a straight line L perpendicular to the geometrical shadow of the object. The point of intersection of the lines L and H is the required point M.

The amplitude of the wave emerging from the point M is chosen equal to

$$A^*(x,y) = -\frac{\left[\,|r_x \cos^2 \psi + r_y \sin^2 \psi|\,\right]^{1/2}}{h^{1/2}\left[(r_x^2 \cos^2 x + r_y^2 \sin^2 \psi)^{1/2}\right]} \exp\left[i\frac{2\pi}{\lambda}P(x,y)\right], \qquad (5)$$

where $r_x = 1/h - 1/h_x, \quad r_y = 1/h - 1/h_y$.

The phase of the wave A^* coincides with the phase of the incident wave (1). The moduli of the amplitudes of these waves are different, but always finite.

Replacing the total action of the secondary Fresnel waves from the elements of the infinite straight line H by the action of one specially chosen wave reduces the two-dimensional diffraction problem to a one-dimensional one. After this, an explanation can be given for the transition phenomenon.

Fig. 2

Fig. 2

Figure 2: Fig. 2

As the observation plane approaches the center of curvature of the normal section C_ψ, the angle between the object and the geometrical shadow approaches a right angle. The angle between the line L and the object tends to zero. The points on the line L corresponding to infinite straight lines not passing through the point B then move farther and farther away from B, and consequently also from the chosen point in the observation plane. In the limit, the action of the waves emerging from these points is equal to zero. This means that the total action of the secondary Fresnel waves from the elements of infinite straight lines not passing through the point B is equal to zero. Only the action of the wave emerging from B remains. Consequently, the light intensity at the chosen point of the observation plane is determined by the action only of those secondary Fresnel waves that are emitted by the elements of the infinite straight line passing through B.

Fig. 3 Fig. 4

Fig. 3. Diffraction pattern from a screen of width 5 mm. $h = 2330$ cm, $h_x = 1165$ cm, $h_y = \infty$, $\psi = 65^\circ$

Fig. 4. Shadow from a screen of width 5 mm, observed in the plane R_ψ. $h = 2330$ cm, $h_x = 1165$ cm, $h_y = \infty$, $\psi = 45^\circ$

If the point B is located outside the object, then all secondary waves from the elements of this straight line arrive at the observation point. The light intensity there is equal to the intensity for free propagation of the wave. If the point B lies in the region occupied by the object, then all secondary waves are diffracted. The intensity at the observation point is equal to zero. The distribution of light intensity is thus described by the laws of geometrical optics.

3. The scheme considered for the transition of a Fresnel diffraction pattern into a geometrical shadow is realized only when the wave is unbounded in the direction of the edge of the object. This is the reason for the absence of diffraction phenomena. The size of the wave in the direction perpendicular to the object is arbitrary.

For a wave bounded in both directions, there will be no geometrical shadow. The distribution of the light intensity will deviate, to one degree or another, from the geometrical one. This deviation can be estimated with the aid of the width of the effective penumbra

$$s_{\text{eff}} = I_\Gamma \left| \left(\frac{\partial I}{\partial l} \right)_C \right|.$$

Here I_Γ is the intensity under free propagation of the wave; l is the direction perpendicular to the geometrical shadow; C is a point at the middle of the boundary of the geometrical shadow.

Under certain limiting conditions

$$s_{\text{eff}} = \lambda/\alpha, \tag{6}$$

where α is the aperture angle (Fig. 1).

4. The experiment was carried out on an apparatus that made it possible to observe diffraction for an incident cylindrical wave. To obtain the wave, a collimator with a focal length of 1917 mm and a cylindrical lens were used. The lens diameter was 200 mm; the width of the collimator slit was 0.02 mm.

The radius of curvature of the converging cylindrical wave emerging from the lens was 1225 cm. At a distance of 60 cm from the lens, an opaque screen 5 mm wide was placed. The diffraction patterns were photographed on film located at a distance of 2330 cm from the screen.

Two values of the angle between the generating line of the wave surface and the edge of the screen were chosen: 25 and 45°. In the first case (Fig. 3), a typical Fresnel diffraction pattern from a narrow screen was obtained. In the second, the plane in which the film is placed passes through the center of curvature of the normal section C_ψ, parallel to the screen; a geometrical shadow is observed (Fig. 4). The width of the effective penumbra under the indicated conditions is 0.06 mm.

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Note: Figure translations are in progress. See original paper for figures.