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ON SHOCK WAVES IONIZING A GAS IN AN ELECTROMAGNETIC FIELD

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This paper considers shock waves that change, from zero to infinity, the electrical conductivity of a gas situated in an arbitrarily oriented magnetic field.* Additional relations are found, whose fulfillment is necessary for the existence of a continuous solution representing the structure of such a wave. These relations do not follow from the conservation laws for mass, momentum, energy, and the continuity of the tangential component of the electric field. They make possible the unique solution of various problems with ionizing waves and the evolutionary character of the waves considered. In the case of a gas with large magnetic viscosity, the additional relations are found explicitly.

The question of a complete system of relations on a shock wave ionizing a gas situated in an electromagnetic field was first considered in papers ([1], [2]), where the magnetic field was assumed to be parallel to the shock-wave front. In paper ([3]), shock waves parallel to the magnetic field were investigated, on both sides of which the conductivity is equal to zero and differs from zero in a narrow layer representing the structure of such a wave (waves with temporary ionization).

Later a number of papers appeared on ionizing shock waves with a magnetic field not parallel to the wave front (see the review ([4]), as well as the recent papers ([5]-[7])); in most cases, the magnetic field ahead of the wave was assumed to be normal to the front. In these papers no attempt was made to find a complete system of boundary conditions on an ionizing shock wave by studying the structure of such a wave. In a formal resolution of the discontinuity conditions, in some cases ([5]) ionizing waves were investigated which, as follows from the present paper, have no structure and, consequently, are not realized. There are also works in which experimental study is carried out of ionizing waves in electromagnetic shock tubes ([4], [8]).

In the present paper the Hall effect is not taken into account. Taking the Hall effect into account cannot change the main conclusions about the number of additional relations on ionizing shock waves of various types and about the evolutionary character of these waves; it can lead only to a change in the form of some of the additional relations.

Let us consider the structure of a shock wave in a gas with allowance for viscosity, thermal conductivity, and electrical conductivity. We shall regard the electrical conductivity of the gas as such a function of the state of the gas, $\sigma = \sigma(\rho, T)$, that in some region of values of ρ, T, in particular ahead of the shock wave, it is equal to zero. The structure of an ionizing shock wave is represented, in the region $\sigma > 0$, by a stationary solution of the magnetohydrodynamic equations, and in the region $\sigma = 0$ by a stationary solution of the gasdynamic equations and the rela-

* A detailed derivation of the results obtained will be published in the jubilee collection dedicated to L. I. Sedov.

with conditions $H_y = \text{const}$, $H_z = \text{const}$. These solutions, at the boundary separating the regions $\sigma > 0$ and $\sigma = 0$, must pass continuously into one another. As $x \to -\infty$ and $x \to \infty$, the quantities constituting the solution must tend to certain limiting values, representing the coordinates of singular points of the corresponding systems of ordinary differential equations in the space of the sought quantities. As is known [9,10], there exist no more than four singular points of the magnetohydrodynamic equations. The gasdynamic singular points form a two-dimensional surface, and their position on this surface is specified by the values of H_y and H_z.

The discontinuity corresponding to the transition from any point of this surface to a magnetohydrodynamic singular point satisfies the conservation laws and occurs at one and the same value of the tangential component of the electric field. Therefore, if only these requirements are imposed as the conditions at the jump, then the shock transition is determined only up to two arbitrary parameters, which specify the position of the gasdynamic singular point. The requirement that a continuous solution representing the structure exist leads, in a number of cases, to additional relations that are not a consequence of the conservation laws and of the continuity of the tangential component of the electric field. The number and character of these relations depend on the type of singular points representing the states ahead of and behind the wave. The character of the singular points, in turn, is determined by the relations between the gas velocity and the propagation velocities of small perturbations. We note that the additional relations mentioned can be written in various forms. In what follows we shall write them either as relations between the changes of the magnetic-field components in the wave and the values of the quantities behind the wave, or as relations between the components of the electric field and the values of the quantities ahead of the wave. The second form of writing is readily obtained from the first by using the relations expressing the conservation laws and the continuity of the tangential component of the electric field. The specific form of the additional relations depends on the ratios of the dissipative coefficients, which are functions of the state of the gas.

Let us consider different types of waves.

1. **Fast supersonic wave.** For it $u_1 > a_0$, $a_A < u_2 < a_+$. Here and below,

$u_1,\ u_2$ are the normal component of the gas velocity ahead of and behind the wave, respectively; a_0 is the speed of sound ahead of the wave, and $a_-,\ a_A,\ a_+$ are the velocities of slow, Alfvén, and fast disturbances behind the wave. For the existence of the structure of such a wave, two additional relations must be satisfied. One of them, in the coordinate system in which $H_{z1} = 0$, is the condition

$$\Delta H_z \equiv H_{z2} - H_{z1} = 0 \tag{1}$$

or

$$E_y = 0. \tag{2}$$

Here $H_y,\ H_z,\ E_y,\ E_z$ are the components of the magnetic and electric fields lying in the plane of the wave, and the subscripts 1 and 2 denote the states ahead of and behind the wave.

In the limiting case when the magnetic viscosity is much greater than the other dissipative coefficients, the second relation has the form

$$\Delta H \equiv H_{y2} - H_{y1} = 0 \quad \text{or} \quad E_z = -\frac{H_{y1}}{c}\, u_2(\rho_1, T_1, H_{y1}). \tag{3}$$

Thus, if a fast supersonic wave propagates through a given state of the gas with a given magnetic field, then the state of the gas and the magnetic field behind the wave, as well as the electric field, are functions only of the shock-wave velocity.

2. **Fast subsonic wave.** For it $u_1 < a_0,\ a_A < u_2 < a_+$. For the existence of the structure of this wave, three additional relations must be satisfied. It turns out that such a wave can propagate into a given state of the gas with a given magnetic field only with a quite definite velocity, and the quantities $\Delta H_y,\ \Delta H_z$, or $E_y,\ E_z$, are determined uniquely. In the limiting case of large magnetic viscosity such a wave is absent.

3. **Intermediate supersonic wave.** On it $u_1 > a_0,\ a_- < u_2 < a_A$. For the existence of the structure of such a wave it is necessary that one additional relation be satisfied, $\Delta H_y = f(\Delta H_z)$. In the case of large magnetic viscosity and small ΔH_z, it has the form

$$\Delta H_y = \left(\frac{\gamma E_0 - H_0^2 H_y}{(1 - a_A^2/u^2)(u^2 - a_0^2) + H_y^2/4\pi\rho} \right)_2 \frac{(\Delta H_z)^2}{2} + O((\Delta H_z)^4), \tag{4}$$

where $E_0 = cE_z/4\pi\rho u,\ H_0 = H_x/4\pi\rho u$.

In the plane case ($H_z = 0$) there exist intermediate waves of two types—those that do not change the magnetic field and those that change the magnitude and sign of the tangential component of the magnetic field.

Thus, if this wave propagates into a given state of the gas with a given magnetic field, then, in order to find all the quantities behind the wave and the electric field, it is necessary, in addition to the shock-wave velocity, also to specify ΔH_z.

4. **Intermediate subsonic wave.** For it $u_1 < a_0$, $a_- < u_2 < a_+$. In this case two additional relations occur; the quantities behind the wave and the electric field are determined if the gas-dynamic quantities and the magnetic field ahead of the wave are known and the wave velocity is specified.

In the case of large magnetic viscosity the additional relations are the equality (4) and the equation

$$\Phi(\rho_1, T_1) = 0, \tag{5}$$

which expresses that the initial parameters belong to the boundary of the region $\sigma > 0$.

In concrete problems, when the gas-dynamic state is arbitrary, ahead of the ionizing wave there will go a gas-dynamic shock wave, the intensity of which is determined by relation (5).

5. **Slow supersonic wave.** On it $u_1 > a_0$, $u_2 < a_-$. On an ionizing wave of this type only the conservation laws are satisfied. To determine the quantities behind the wave and the electric field, it is necessary to specify, in addition to the values of the quantities ahead of the wave and the wave velocity, the change of both components of the magnetic field ΔH_y, ΔH_z from a certain region of admissible values. It is also possible to prescribe both components of the electric field arbitrarily, and to determine ΔH_y, ΔH_z.

6. **Slow subsonic wave.** On it $u_1 < a_0$, $u_2 < a_-$. In this case one additional relation occurs. The quantities behind the wave and the electromagnetic field are determined if the values of the quantities ahead of the wave are known and the wave velocity and ΔH_z are specified. In the case of large magnetic viscosity the additional relation will be (5). Here, as in the case of the subsonic intermediate wave, the state ahead of the ionizing wave is not arbitrary; rather, a gas-dynamic shock wave of definite intensity can propagate ahead of it.

In addition to the ionizing shock waves considered above, waves with temporary ionization are also possible, i.e., such that the conductivity is nonzero only in the transition layer and is equal to zero ahead of and behind the shock wave. The state of the gas behind such waves is always subsonic. For $u_1 > a_0$, for the existence of the structure of such a wave it is necessary that two additional relations be satisfied, and for $u_1 < a_0$—three.

As is well known ([11]), the necessary condition for evolutionarity is that the number of different waves of small perturbations diverging from the discontinuity be one less than the number of conditions at the shock wave. It is easy to verify that, with allowance for the additional relations, all the ionizing shock waves considered are evolutionary.

Note added in proof. After the present work had been submitted for publication, a paper by R. T. Taussig ([12]) appeared, devoted to the same topic. From the references in that paper the authors learned of the work of M. D. Cowley ([13]). The papers of R. T. Taussig and M. D. Cowley contain some of the results published in the present article; however, for consistency of exposition the authors decided not to change the plan of their article. The authors express their gratitude to Prof. R. A. Gross (USA), who, during his stay in the USSR, drew their attention to R. T. Taussig' s work.

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Note: Figure translations are in progress. See original paper for figures.