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Abstract

Full Text

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The electrical conductivity of heterogeneous media consisting of two components is well described by V. I. Odelevskii' s formula (1), if the electrical conductivities of the components differ by no more than 2 orders of magnitude. If the electrical conductivities of the components differ by more than 2 orders of magnitude, then the smaller value of the electrical conductivity is not taken into account by the indicated formula, and the formula does not correspond to the experimental data, as is seen from Figs. 1 and 2.

Suppose that in one particle of the first component, whose electrical conductivity is γ_1, the electromagnetic power losses are

$$P_1 = \gamma_1 E^2 V_1, \tag{1}$$

where P_1 is the electromagnetic power loss, and V_1 is the volume of one particle.

Fig. 1. Dependence of the electrical conductivity of a mixture of two substances ($\gamma_1 = 10^{-2}$ ohm$^{-1} \cdot$ m^{-1}; $\gamma_2 = 10^{-7}$ ohm$^{-1} \cdot$ m^{-1}) on the concentration of the mixture, calculated by Odelevskii' s formula (1)

Fig. 2. Dependence of the losses of electromagnetic energy on the concentration of a mixture of two substances ($\gamma_1 = 10^{-2}$ ohm$^{-1}\cdot$m^{-1}; $\gamma_2 = 10^{-7}$ ohm$^{-1}\cdot$m^{-1}). 1 –experimental dependence; 2 –dependence calculated by formula (5); 3 –dependence calculated by formula (7)

Fig. 2. Dependence of the losses of electromagnetic energy on the concentration of a mixture of two substances ($\gamma_1 = 10^{-2}$ ohm$^{-1} \cdot$ m^{-1}; $\gamma_2 = 10^{-7}$ ohm$^{-1} \cdot$ m^{-1}). 1 —experimental dependence; 2 —dependence calculated by formula (5); 3 —dependence calculated by formula (7).

Figure 2: Fig. 2. Dependence of the losses of electromagnetic energy on the concentration of a mixture of two substances ($\gamma_1 = 10^{-2}$ ohm$^{-1} \cdot$ m^{-1}; $\gamma_2 = 10^{-7}$ ohm$^{-1} \cdot$ m^{-1}). 1 —experimental dependence; 2 —dependence calculated by formula (5); 3 —dependence calculated by formula (7).

Let the volume V contain n_1 particles of the first component; then the total losses in the particles of the first component will be

$$P_{1n_1} = \gamma_1 E^2 V_1 n_1. \tag{2}$$

The total volume of the first component may be related to the content δ_1 of the first component in the volume V:

$$P_1 = \gamma_1 E^2 V \delta_1. \tag{3}$$

Since the electrical conductivity of the first component is higher by 2 or more orders of magnitude than that of the second, the losses in the second component may be neglected.

Let us consider two possible cases:

a) The content δ_1 is small, and the influence of the Lorentz local field of one particle on neighboring particles is negligible. In this case, assuming that the power P_{1n_1} is dissipated in the volume V of the heterogeneous medium with electrical conductivity σ,

$$P_1 = \sigma E^2 V, \tag{4}$$

comparing (3) and (4), we obtain the electrical conductivity of the given medium as a function of the first component,

$$\sigma = \gamma_1 \delta_1. \tag{5}$$

b) With a significant content of the first component, it is necessary to take the Lorentz field into account, i.e., to set

$$E_{\mathrm{L}} = \frac{\varepsilon + 2}{3} E_{\mathrm{av}}, \tag{6}$$

after which, substituting (6) into (3) and taking (4) into account, we obtain

$$\sigma = \gamma_1 \delta_1 \left(\frac{\varepsilon + 2}{3} \right)^2 . \tag{7}$$

An experimental verification of expressions (5) and (7), as well as determination of the limits of their applicability, was carried out as follows: the heterogeneous medium was placed in a section of a coaxial line of length L and conductor radii r_1 and r_2. The power dissipated in this section was measured as a function of the content of the first component. Figure 2 shows the typical dependence $P = f(\delta_1)$ obtained in the experiment. The losses in this section of the line are described by the expression

$$P = \frac{\pi \sigma u_0^2}{\alpha \ln(r_2/r_1)} [1 - \exp(-2\alpha L)], \tag{8}$$

where α is the attenuation coefficient of the electromagnetic wave, expressed by the well-known formula

$$\alpha = \omega \sqrt{\frac{\varepsilon_a \mu_a}{2} \left[-1 + \sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon_a} \right)^2} \right]}, \tag{9}$$

where σ is determined by formula (5) or (7).

The calculated dependences $P = f(\delta_1)$ for functions (5) and (7) are shown in Fig. 2. As can be seen, the linear approximation (5) is well justified for values of δ_1 not exceeding 0.4–0.5, whereas expression (7), which takes the local fields into account, is justified for values greater than 0.4–0.5.

Thus, the electrical conductivity of a heterogeneous medium in which the conductivities of the components differ by more than 2 orders of magnitude, for contents of the component with the higher electrical conductivity up to 0.4–0.5, is described by expression (5), which does not take the Lorentz field into account, and for contents greater than 0.4–0.5 by expression (7), which takes this field into account.

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