GREEN' S FUNCTIONS FOR QUANTUM FERMI SYSTEMS OF CHARGED PARTICLES WITH COLLISIONS

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Abstract

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GREEN' S FUNCTIONS FOR QUANTUM FERMI SYSTEMS OF CHARGED PARTICLES WITH COLLISIONS

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Charged Fermi particles are considered, interacting according to Coulomb' s law and situated in a compensating field of charge of the opposite sign. We shall start from equations for quantum distribution functions of the form

$$D_1(t, x_1, x_1') = \langle \Psi^+(t, x_1');\ \Psi(t, x_1) \rangle,$$

where $\langle ... \rangle = Q^{-1}\,\mathrm{Sp}(...\, e^{-(H-\lambda N)/\theta})$; $Q = \mathrm{Sp}\, e^{-(H-\lambda N)/\theta}$, λ is the chemical potential; $x = \{\bar{r}, \sigma\}$,

$$\Psi(t, x) = \frac{1}{(2\pi)^{3/2}} \int e^{ipx} a_{p\sigma}\, dp, \qquad \Psi^+(t, x) = \frac{1}{(2\pi)^{3/2}} \int e^{-ipx} a^+_{p\sigma}\, dp.$$

Then

$$D_1(t, x_1, x_1') = \frac{1}{(2\pi)^3} \iint e^{i(p_1 x_1 - p_1' x_1')} \langle a^+_{p_1' \sigma_1'}, a_{p_1 \sigma_1} \rangle\, dp_1\, dp_1' =$$

$$= \frac{1}{(2\pi)^3} \iint e^{i(p_1 x_1 - p_1' x_1')} F(p_1, \sigma_1, p_1' \sigma_1')\, dp_1\, dp_1'.$$

The equilibrium value is

$$F^0(p_1, p_1') = n_{p_1} \delta(p_1 - p_1') \delta(\sigma_1 - \sigma_1'), \qquad n_p = 2/(e^{(\varepsilon - \lambda)/\theta} + 1),$$

$$\varepsilon = \hbar^2 p^2 / 2m, \qquad D_1^0(x_1, x_1') = \frac{1}{(2\pi)^3} \frac{\delta(\sigma_1 - \sigma_1')}{2} \int e^{ip_1(x_1 - x_1')} n_{p_1}\, dp_1.$$

The functions D, in contrast to Bogolyubov distribution functions ([1]), are normalized to n, and not to 1. Let us take the kinetic equation with a self-consistent term in the Hartree approximation

$$i\hbar\frac{\partial D_1(t,x_1,x_1')}{\partial t}+\frac{\hbar^2}{2m}(\Delta_{r_1}-\Delta_{r_1'})D_1(t,x_1,x_1')-$$

$$-D_1(t,x_1,x_1')\int[\Phi(r_1-r_2')-\Phi(r_1'-r_2')]D_1(t,x_2',x_2')\,dx_2'=I_{\rm st}. \qquad (1)$$

The case $I_{\rm st}=0$ was considered in work ([2]). We linearize equation (1) and perform a symmetric Fourier transformation with respect to spatial variables of the type

$$F\{...\}=\frac{1}{(2\pi)^3}\int\{...\}e^{-ikr+ik'r'}\,dr\,dr'. \qquad (2)$$

As a result we obtain the equation in k-space

$$\left[i\hbar\frac{\partial}{\partial t}-\frac{\hbar^2}{2m}\left(k_1^2-k_1'^2\right)\right]\delta F(k_1k_1')-$$

$$-\frac{\Delta(\sigma_1-\sigma_1')}{2(2\pi)^{3/2}}\rho(k_1-k_1')\Phi(k_1-k_1')\left(n_{k_1'}-n_{k_1}\right)=I_{\rm st}^{\rm lin} \qquad (3)$$

where

$$\rho(q)=\sum_{\sigma_2}\int\delta F(k,\sigma_2,k-q,\sigma_2)\,dk;\qquad p_1-p_1'=q,\quad p_1=k.$$

Let us imagine that the system is placed in an infinitesimally small external field and that the variation δF is due precisely to this field. Let

$$\delta H=e^{-iEt}\delta\Omega(x_1,x_2')/2\pi,$$

then (3) becomes

$$i\hbar\frac{\partial\delta F(t,k_1,\sigma_1,k_1',\sigma_1')}{\partial t}-\frac{\hbar^2}{2m}(k_1^2-k_1'^2)\delta F(t,k_1,\sigma_1,k_1',\sigma_1')-$$

$$-\frac{\delta\Omega(t,k_1,\sigma_1,k_1',\sigma_1')}{4\pi}\left(n_{k_1'}-n_{k_1}\right)-$$

$$-\frac{\Delta(\sigma_1-\sigma_1')}{2(2\pi)^{3/2}}\rho(k_1-k_1')\Phi(k_1-k_1')\left(n_{k_1'}-n_{k_1}\right)=I_{\mathrm{st}}^{\mathrm{lin}}. \tag{4}$$

We apply the theorem on variations of the mean value of a dynamical quantity (see ([3]), and for the classical case ([2,4]))

$$\delta F(t,k_1,k_1',\sigma_1,\sigma_1')/\delta\Omega(t,p,p',\sigma,\sigma')=G^{\mathrm{ret}}(E;k_1,k_1',\sigma_1,\sigma_1';p,p',\sigma,\sigma'), \tag{5}$$

$$\delta\Omega(t,k_1,k_1',\sigma_1,\sigma_1')/\delta\Omega(t,p,p',\sigma,\sigma')=\delta(k_1-p)\delta(k_1'-p')\Delta(\sigma-\sigma_1)\Delta(\sigma_1'-\sigma'). \tag{6}$$

Then for the Green' s function we obtain the equation

$$\hbar E G_E(k_1,k_1',\sigma_1,\sigma_1';k,k',\sigma,\sigma')-\frac{\hbar^2}{2m}(k_1^2-k_1'^2)G_E-$$

$$-\frac{\Delta(\sigma_1-\sigma_1')}{2(2\pi)^{3/2}}\Phi(k_1-k_1')\rho(k_1-k_1';k,k',\sigma,\sigma')\left(n_{k_1'}-n_{k_1}\right)-$$

$$-\frac{\Delta(\sigma-\sigma_1)\Delta(\sigma'-\sigma_1')}{4\pi}\delta(k-k_1)\delta(k'-k_1')\left(n_{k_1'}-n_{k_1}\right)=I_{\mathrm{st}}^{G},$$

$$\rho(q;k,k',\sigma,\sigma')=\sum_{\sigma_1}\int G_E(k_1,k_1-q,\sigma_1,\sigma_1;k,k',\sigma,\sigma')\,dk_1. \tag{7}$$

It remains to choose the collision integral and to write it in terms of Green' s functions. The quantum collision integral was obtained in works ([5,6]), etc. Let us consider the expression obtained by Wyld and Fried ([7]) for the spatially homogeneous case and for particles without spin,

$$I_{\mathrm{st}}=-\frac{i}{(2\pi)^2}\iint dp'\,dq'\left|\frac{\Phi(q')}{K\left\{q',\frac{1}{\hbar}(E_{p+q'}-E_p)\right\}}\right|^2\times$$

$$\times\delta(E_{p+q'}+E_{p'-q'}-E_p-E_{p'})\left\{F(p)F(p')[1-F(p+q')]\right\}\times$$

$$\times[1-F(p'-q')]-F(p+q')F(p'-q')[1-F(p)][1-F(p')]\Big\}, \tag{8}$$

where

$$K(q,\Omega) = 1 + \frac{\Phi(q)}{(2\pi)^{3/2}} \int \frac{F(p''+q/2) - F(p''-q/2)}{\hbar\Omega + E_{p''-q/2} - E_{p''+q/2} + i\eta}\, dp'', \tag{9}$$

$$E_p = \hbar^2 p^2/2m, \qquad \eta > 0, \quad \eta \to 0.$$

Let us recall that this integral was obtained under the assumption that the higher distribution functions depend on time only through the first func-

i.e., the equation is valid for $t > \tau_{\text{coll}}$. Moreover, I_{st} is proportional to the small parameter $\varepsilon = 8\pi e^2 m/r_{dp}F^2$,

$$r_d = (8\pi e^2 mn/p_F^2)^{-1/2}, \qquad n = N/V, \qquad p_F = (3\pi^2 n)^{1/3}\hbar,$$
$$\varepsilon = \sqrt{\pi}/3(8e^2 m/\hbar p_F)^{3/2} = \sqrt{\pi}/3\sigma^{3/2} \ll 1. \tag{10}$$

Consequently, in the present theory ε, or $\sigma = 8e^2 m/p_F\hbar$, are small parameters. Since $\varepsilon = (8\pi e^2 m/\hbar^2)^{3/2}/3\pi^2 n^{1/2}$, it is clear that $\varepsilon \ll 1$ for sufficiently dense systems at low temperatures. Let us calculate the collision integral (8) and assume that the linearized equation obtained is also valid for a spatially inhomogeneous system.

Applying the variational theorem to (5), (6), we find

$$\begin{aligned} I_{\text{st}}^G = -\frac{i}{(2\pi)^2} \iint dp'\, dq\, Q^0(q,k_1)\, \delta(E_{k_1+q} + E_{p'-q} - E_{k_1} - E_{p'}) \times \\ \times \{G_E(k_1,k_1',\sigma_1,\sigma_1';k,k',\sigma,\sigma')s_1 + G_E(p',k_1',\sigma_1,\sigma_1';k,k',\sigma,\sigma')s_2 - \\ - G_E(k_1+q,k_1',\sigma_1,\sigma_1';k,k',\sigma,\sigma')s_3 - \\ - G_E(p'-q,k_1',\sigma_1,\sigma_1';k,k',\sigma,\sigma')s_4\}, \end{aligned} \tag{11}$$

where

$$Q^0(q,k_1) = \left| \frac{\Phi(q)}{K^0(q,(1/\hbar)(E_{k_1+q} - E_{k_1}))} \right|^2;$$

$$\begin{aligned} s_1 &= n_{p'} - n_{p'}n_{k_1+q} - n_{p'}n_{p'-q} + n_{k_1+q}n_{p'-q}; \\ s_2 &= n_{k_1} - n_{k_1}n_{k_1+q} - n_{k_1}n_{p'-q} + n_{k_1+q}n_{p'-q}; \\ s_3 &= n_{p'-q} + n_{k_1}n_{p'} - n_{p'-q}n_{k_1} - n_{p'-q}n_{p'}; \\ s_4 &= n_{k_1+q} + n_{k_1}n_{p'} - n_{k_1+q}n_{k_1} - n_{k_1+q}n_{p'}, \end{aligned}$$

and K^0 means that n_p has been substituted in place of F.

The solution of equation (7), which for simplicity we consider for spinless particles, with I_{st}^{G} in the form (11), will symbolically be

$$G = \frac{L + \mathcal{L}}{\hbar E - (\hbar^2/2m)(k_1^2 - k_1'^2) + iM}, \tag{12}$$

where

$$M = \frac{1}{(2\pi)^2} \iint dp'\, dq\, Q^0(q, k_1) s_1 \delta(E_{k_1+q} + E_{p'-q} - F_{k_1} - E_{p'}),$$
$$L = \frac{1}{(2\pi)^{3/2}} \rho(k_1 - k_1'; k, k') \Phi(k_1 - k_1')(n_{k_1'} - n_{k_1}) +$$
$$+ \frac{1}{2\pi} \delta(k - k_1) \delta(k_1' - k')(n_{k_1'} - n_{k_1}), \tag{13}$$
$$\mathcal{L} = I_{\text{st}}^{G} - iMG_E.$$

The structure of expression (12) is analogous to the classical expression for the Green's function. Therefore here, too, one may neglect $\mathcal{L}$ with high accuracy for values of the wave numbers much smaller than k_F. Let us show this by passing to the dimensionless k/k_F and q/k_F. $L \sim e^2/q^2$. Q^0 will be $\sim me^4/k_F^3$. In order not to take $\mathcal{L}$ into account, the condition $\mathcal{L} \ll L$ must be fulfilled; hence $q^2 \ll k_F^2/\sigma$, i.e., for $q < k_F$ and even $q \sim k_F$, since $\sigma \ll 1$.

Therefore, for $q < k_F$ we have

$$G_E = L/[\hbar E - (\hbar^2/2m)(k_1^2 - k_1'^2) + iM]. \tag{14}$$

Introduce the substitution $k_1' + q = k_1, \quad k_1 = k_1$. Then (14) becomes

$$[\hbar E - (\hbar^2/2m)(2k_1 - q)q + iM(k_1)] G_E(k_1, k_1 - q; k, k') -$$
$$-(2\pi)^{-3/2} \rho(q; k, k') \Phi(q)(n_{k_1-q} - n_{k_1}) =$$
$$= (2\pi)^{-1} \delta(k - k_1) \delta(k' + q - k_1)(n_{k_1-q} - n_{k_1}). \tag{15}$$

Next, after simple operations of division and integration, we arrive at the following solution for the Green' s functions:

$$\rho(q; k, k') = \frac{1}{2\pi} \frac{(n_{k'} - n_k)\delta(k' + q - k)}{\hbar E - (\hbar^2/2m)(2k - q)q + iM(k)} \times \left[1 + \frac{\Phi(q)}{(2\pi)^{3/2}} \int \frac{(n_k - n_{k-q})\, dk}{\hbar E - (\hbar^2/2m)(2k - q)q + iM(k)}\right]^{-1}. \tag{16}$$

Thus, in contrast to work ([2]), here the damping of the system due to collisions is taken into account. Damping without allowance for polarization by the method of kinetic equations was considered in work ([8]).

Knowing the function ρ, with the aid of the spectral theorems ([3]) it is not difficult to calculate the thermodynamic characteristics of the system, as well as the correlation functions. For this it is necessary to find the spectral density I from the formula

$$I(E) = -2\,\mathrm{Im}\,G_E/(e^{E/\theta}+1).$$

After simple calculations we obtain

$$\begin{aligned} I(E) = &-\delta(k-k'-q)(n_{k'}-n_k)\{M(k)\,\mathrm{Re}\,K^0+\\ &+[\hbar E-(\hbar^2/2m)(2k-q)q]\,\mathrm{Im}\,K^0\}\times\\ &\times\{2\pi(e^{E/\theta}+1)(\mathrm{Re}^2\,K^0+\mathrm{Im}^2\,K^0)[[\hbar E-\\ &-(\hbar^2/2m)(2k-q)q]^2+M^2(k)]\}^{-1}. \end{aligned}$$

The damping and frequencies of the system are determined as the poles of the Green's function (16). For this purpose, the values K^0 and M are first calculated from formulas (9) and (13). In the case of small values of the wave numbers at $\theta = 0$,

$$Q^0(q) = \frac{2e^4}{\pi(q^2+4ck_F)^2}, \qquad c = \frac{me^4}{2\pi\hbar^2} \sim \sigma.$$

For $\theta = 0$, and also for small values of the wave numbers, the frequencies coincide with the results of the works of V. P. Silin; the collisional damping is absent for $k \ll k_F$. It appears only in a term $\sim (k/k_F)^2$, which corresponds to the result of Dubois ([9]).

Analogous results are also obtained for $\theta \ll \varepsilon_F = \mu_0$, with the values $Q^0(q)$ differing from the case $\theta = 0$ only by a change of the factor

$$c^* \to c: \qquad c^* = c\left\{1-\frac{\pi^2}{12}\left(\frac{\theta}{\mu_0}\right)^2\right\}.$$

It should be noted that the exact limits of applicability of equation (11) can be estimated only on the basis of a more general inhomogeneous quantum kinetic equation with collisions. For the present one may say that it is applicable for times in the interval $\tau_{\mathrm{int}} < t < \tau_{\mathrm{rel}}$, where τ_{rel} is determined by the damping of the system, and also in the interval of wave vectors $q < k_F$.

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Note: Figure translations are in progress. See original paper for figures.