ON THE POSSIBILITY OF ESTIMATING THE SIZES AND MASSES OF THE ICY NUCLEUS OF A COMET

Astronomy

1968

SovietRxiv

Abstract

Full Text

UDC 523.66

Astronomy

D. O. Mokhnach

ON THE POSSIBILITY OF ESTIMATING THE SIZES AND MASSES OF THE ICY NUCLEUS OF A COMET

(Presented by Academician B. P. Konstantinov, 24 VII 1967)

1. On the photodissociation of parent molecules

Let us suppose that, upon absorption of a quantum $h\nu$, dissociation of a parent molecule occurs. The decay products are the molecule observed by us, with mass m_1 and velocity v_1, and a residue with mass m_2 and velocity v_2. By the law of conservation of momentum,

$$m_1 v_1 - m_2 v_2 \sim 10^{-22}, \tag{1}$$

if the dissociation occurs upon absorption of quanta in the visible part of the solar spectrum. Since the molecular masses are of order 10^{-22}, and the velocities $v \sim 10^5$, one may take

$$m_1 v_1 = m_2 v_2. \tag{2}$$

If the dissociation energy of the parent molecule is $E_d = h\nu_0$, then by the law of conservation of energy

$$h\nu = h\nu_0 + {}^1/_2 m_1 v_1^2 (1 + m_1/m_2). \tag{3}$$

Let us assume, for example, that the dissociation energy of the parent molecule producing C_2 is $h\nu_0 = 2$ eV, i.e. absorption of quanta with $\lambda \leqslant 6200$ Å leads to the appearance of decay products with v_1 and $v_2 \geqslant 0$. Let $m_2 = 4m_1$. Then, in order to obtain a velocity $v_1 = 3 \cdot 10^5$ cm/sec for the C_2 molecule, with mass $m_1 = 0.4 \cdot 10^{-22}$ g, absorption of a quantum with $\lambda \sim 3700$ Å is sufficient.

For $\lambda < 6200$ Å at a distance of 1 AU from the Sun,

$$\Phi_\lambda \sim 10^{17} \text{ photons/cm}^2 \cdot \text{sec},$$

and for $\lambda \leqslant 3700$ Å,

$$\Phi_\lambda \sim 2 \cdot 10^{16} \text{ photons/cm}^2 \cdot \text{sec}.$$

As is known, for typical comets (1942 g, 1959 k, and similar ones) the emission coefficient, found under the assumption of an isotropic model with an emission center, is

$$n_0 \sim 10^{27} \text{ molecules/sec} \cdot \text{steradian}$$

for CN and C_2 molecules. In bright comets, n_0 may be considerably larger and reach values of 10^{30}–10^{31}.

Consequently (in our example), in order to attain such a value of the emission coefficients, it is necessary to assume that the surface from which C_2 and CN are emitted is

$$10^{10} < \pi r^2 < 10^{14} \text{ cm}^2.$$

From this one can estimate the radius of the cloud of parent molecules:

$$10^5 < r < 10^7 \text{ cm}.$$

The lifetime of the parent molecules, whose nature is completely unclear to us, is apparently very short. If the effective cross section for photodis–

dissociation of the parent molecule is even taken to be equal to 10^{-16} cm^2 (i.e., the value we obtained for C_2), then

$$\tau = 1/\Phi_\lambda \sigma_d \sim 10^{-1} \text{ sec}.$$

Consequently, at any velocities of emission of parent molecules from the nucleus, photodissociation occurs in the immediate vicinity of the nucleus. The value of r obtained above may be attributed to the solid nucleus of the comet, and one may assume that the radius of the nucleus of a typical comet is $r \sim 1$ km, while in bright comets it may reach tens of kilometers.

It follows from this that, in order to maintain the constant brightness of a comet, emission from the surface of the nucleus is required:

$$N \sim 10^{17} \text{ molecules/sec} \cdot \text{cm}^2.$$

P. Egibekov ([1]) considered the process of evaporation of the snowy cover of a cometary nucleus and found the evaporation coefficient at a distance of 1 AU:

$$z_0 = 4.5 \cdot 10^{17} \text{ molecules/cm}^2 \cdot \text{sec.}$$

The velocities of the molecules are measured in tens of meters per second.

P. Egibekov' s calculations were made under the assumption that the evaporation of free radicals proceeds from the surface of the solid nucleus of a rotating comet; the radicals adhere to the nucleus at large distances from the Sun by condensation. The small escape velocity of molecules under the hypothesis to which P. Egibekov adheres does not allow one to explain the observed distribution of surface brightness in the head of a comet.

The hypothesis of parent molecules appears much more plausible in the present case. The preliminary value we have adopted,

$$E_d = 2 \text{ eV},$$

for the parent molecules is quite probable. It is consistent with the supposition that the weak emissions observed ([2]) in comets at $\lambda\lambda$ 7088, 7381, 8242, 8783 Å, hitherto unidentified, may be attributed to parent molecules. P. Swings also does not object to this.

If the nucleus consists of ices emitting parent molecules, then the cause of the appearance of a cloud of parent molecules in the vicinity of the nucleus may be the destruction of ice crystals by the infrared radiation of the Sun.

The total number of photons in the region from λ 10 000 Å to λ 20 000 Å is

$$\Phi_\lambda \sim 1.5 \cdot 10^{17} \text{ photons/cm}^2 \cdot \text{sec.}$$

To obtain an emission coefficient $n_0 \sim 10^{27}$—10^{28} molecules/sec · steradian, a nucleus with radius $r \sim 10^5$ cm is required (for faint comets), while for bright comets with $n_0 \sim 10^{30}$—10^{31} the radius of the nucleus must reach several tens of kilometers. In this case, the practically instantaneous destruction of the parent molecules upon absorption at $\lambda \leq 6000$ Å will give us the necessary number of luminous molecules (radicals) with high velocities.

If half the energy of the absorbed photon is expended on destroying the crystal, then the velocity of a parent molecule with mass $m \sim 10^{-22}$ g may reach 10^5 cm/sec. But taking into account that the lifetime of the parent molecules is measured in fractions of a second, we must come to the conclusion: the radius of the cloud of parent molecules is practically equal to the radius of the solid icy nucleus of the comet.

2. Estimate of the radius of the solid nucleus of a comet. The nucleus has not yet been observed by the instruments that have been used for photographic or visual study of comets. This is understandable if one assumes that the nucleus is of small radius, shining by reflected—

···by sunlight, is lost against the background of the intense resonance radiation of C_2, CN, and other constituents of the comet.

For example, for a nucleus radius $R = 18 \cdot 10^5$ cm at a distance $\Delta = 1$ AU and with an albedo equal to the lunar one, the stellar magnitude of the nucleus is $m = 10^m.5$, while for an albedo of 0.4–0.5, $m \sim 8^m.5$. The emission coefficient from the hemisphere in this case is somewhat greater than 10^{30} molecules/sec. Such an emission coefficient is observed in bright comets 1956 n Arend–Roland or 1957 d Mrkos, whose stellar magnitude reached 1^m. In these comets, intense CN emissions were observed with the greatest surface brightness near the nucleus. Against such a bright background, a nucleus with a diameter of $1''$ and a brightness of 8^m–9^m, of course, cannot be observed by the usual photographic method. It should be noted that an emission coefficient of 10^{30} is sufficient, according to A. Z. Dolginov' s hypothesis ([3]), for the formation of condensation centers and the appearance of the dust component in the head of the comet.

For some comets, such as Encke–Backlund' s comet, the question of the size of the nucleus can be clarified on the basis of the secular decrease in brightness. As S. K. Vsekhsvyatskii showed ([4]), the brightness of Encke' s comet decreased by almost 2 over 50 revolutions. If we suppose that the surface brightness varies according to the law

$$N \sim n/r,$$

where n is the emission coefficient, then when n decreases it is necessary to reduce r–the distance to the photometric center–by the same factor, so that at the edge of the image the same limiting value N_1, perceived by the instrument, is preserved.

In this case the brightness of the comet, to within a constant factor, can be calculated by the formula

$$J = n \int_0^{2\pi} \int_0^{r} \frac{1}{\rho}\, \rho \, d\varphi \, d\rho = 2\pi n r. \tag{4}$$

Let us suppose that the brightness of the comet decreased over 50 revolutions by 1^m. Then

$$J_1 = J_0/2.5.$$

Since the emission coefficient n is proportional to the surface of the nucleus S, on the basis of formula (4) we must assume that, when J_0 decreases by 1^m, the surface must decrease by $\sqrt{2.5} = 1.6$ times, i.e.

$$R_1^2 = 0.62 R_0^2,$$

and the radius of the nucleus

$$R_1 = 0.8R_0.$$

Let us denote by α the number of parent molecules emitted per second upon absorption of solar radiation by 1 cm^2 perpendicular to the radius vector. The rate of decrease of mass is

$$dM/dt = -\alpha S/4$$

or

$$4\pi R^2\delta\, dR = -\alpha\pi R^2 dt,$$

where δ is the density of the material of the nucleus. Hence

$$R = C - \frac{\alpha}{4\delta}\, t,$$

and under the initial conditions $R = R_0$ at $t = 0$

$$R_0 - R = \frac{\alpha}{4\delta}\, t. \tag{5}$$

Let us estimate the radius of the icy nucleus of Encke' s comet, assuming a decrease in brightness by 1^m over 50 revolutions. Let us calculate the loss of mass in 1 revolution. Let

α is the mass loss from one square centimeter per second at a distance of 1 AU.

During a time Δt, at a distance r from the Sun, the loss is

$$\Delta M = \frac{\alpha}{r^2}\Delta t,$$

where r is expressed in astronomical units.

Using the expression for the sectorial velocity

$$\frac{1}{2}r^2 d\varphi/dt = \pi a^2\sqrt{1-e^2}/T,$$

where a is the semimajor axis in astronomical units, e is the eccentricity, and T is the period in seconds, we obtain

$$\Delta M = \frac{\alpha T}{2\pi a^2\sqrt{1-e^2}}\, d\varphi.$$

Integrating the right-hand side with respect to φ from 0 to 2π, we find the mass loss over one period

$$\delta M = \alpha T / a^2 \sqrt{1-e^2}. \tag{6}$$

For Encke' s comet, $T = 10^8$ sec, $a = 2.2$, $e = 0.85$, $\alpha = 10^{17}$ molecules/cm^2 sec $= 10^{-5}$ g/cm^2 sec. Then, by formula (6), per one revolution

$$\delta M = 4 \cdot 10^2 \text{ g/cm}^2,$$

and over 50 revolutions

$$\delta M = 2 \cdot 10^4 \text{ g/cm}^2.$$

As shown above, for a loss of brightness by 1^m it is necessary that the comet have $R = 0.8R_0$. Therefore,

$$0.2R_0 = 2 \cdot 10^4 / 4\delta.$$

For density $\delta = 1$,

$$R_0 = 2.5 \cdot 10^4 \text{ cm}.$$

The mass of the comet is $M_0 = 6 \cdot 10^{13}$ g; moreover, over 50 revolutions the comet loses 60% of its mass while its brightness decreases by 1^m. If the secular decrease in brightness is greater, then the initial radius must be smaller. The comet' s nucleus, with the dimensions we have found, must have, depending on albedo, a brightness of 16^m-18^m, which, of course, is inaccessible to observation against the bright background of the comet' s head.

Main Astronomical Observatory
of the Academy of Sciences of the USSR

Received
26 VI 1967

REFERENCES

1. P. Egibekov, Bull. Inst. Astrophysics, Academy of Sciences of the Tajik SSR, No. 48 (1966).

2. P. Swings, L. Haser, *Atlas of Representative Cometary Spectra*, 1956.

3. A. Z. Dolginov, *Astron. Zh.*, No. 2 (1967).

4. S. K. Vsekhsviatskii, *Russk. astr. zh.*, 4, 4 (1927).

Note: Figure translations are in progress. See original paper for figures.