ON THE INFLUENCE OF IMPERFECT FIXING OF THE EDGE OF A SHELL ON LOSS OF STABILITY

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Abstract

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Strictly convex shells used in actual structures are usually rigidly clamped along the edge. In the ideal case this is assumed to mean that the points of the edge and the tangent planes to the surface of the shell at these points are fixed. In reality these conditions are difficult to observe, just as they are difficult to control. The fixing may be insufficiently rigid or imperfect in the sense that the middle surface of the shell is stressed before loading. As a result, the stressed state arising in the shell under the action of the prescribed load differs to one degree or another from that expected, and the true critical value of the load proves to be lower than the theoretical one.

In the present note we consider the question of the extent to which the critical external pressure on a shallow strictly convex shell can be reduced because of imperfect fixing of its edge. At the same time, naturally, we must somehow restrict the imperfection of the fixing, since without this the formulation of the question is meaningless. The initial stressed state of the shell, caused by imperfection in the fixing of the edge, can reduce the magnitude of the critical pressure to any value, even make it negative. We shall restrict the admissible imperfections in the fixing of the shell by the character of the stressed state in the middle surface of the shell at the moment of loss of stability. Namely, we shall require that in the middle surface of the shell, at least at the moment of loss of stability, the normal stresses in all directions be compressive.

Under this assumption, independently of the nature of the imperfections in the fixing of the shell, we shall indicate a simple lower estimate for the magnitude of the external critical pressure, i.e., the pressure at which the shell loses stability and begins to bulge.

In the author’ s book (1) the question of loss of stability of a strictly convex shell under the action of a prescribed system of forces acting along its edge was considered. It was assumed that the shell is sufficiently shallow, that the principal curvatures and principal directions on the middle surface vary little, and, finally, that the stresses σ and τ, characterizing the stressed state of the middle surface at the moment of bulging, vary little. With regard to the fixing of the edge of the shell it was assumed only that it excludes displacements normal to the surface of the shell. The method set forth in the book for solving this problem is readily transferred to the case when the shell is loaded not only along the edge, but also over the whole surface by a uniformly distributed external pressure p.

In this more general formulation the functional $W = U - A - A(p)$, to the consideration of which the problem of loss of stability is reduced, differs from the functional $W = U - A$ in the problem considered by the term $A(p)$, which represents the work performed by the external pressure p during bulging of the shell. This work for one region of bulging is determined by the formula

$$A_1(p) = \pi p \sigma \sqrt{R_1 R_2}\,(\lambda^4 + \mu^4 + 4\lambda^2\mu^2).$$

In the case $\vartheta = 0$ the derivation of this formula is given in the book (2).

From the condition of stationarity of the functional W at the moment of buckling, with respect to the load parameters and the parameters λ, μ, ϑ, characterizing the dimensions and orientation of the buckling regions, one now obtains the relation

$$\frac{2E\delta^2}{\sqrt{3}(1-\nu^2)R_1R_2} \leq \left\{ \left(\frac{2\delta\sigma_1}{R_1} - \frac{2\delta\sigma_2}{R_2} \right)^2 + \frac{16\delta^2}{R_1R_2}\tau^2 \right\}^{1/2} + p. \tag{*}$$

For $p = 0$ it becomes the inequality obtained in the book (1). Here R_1 and R_2 are the principal radii of curvature of the shell; δ is its thickness; E and ν are Young' s modulus and Poisson' s ratio; σ_1, σ_2, and τ are the stresses in the middle surface at the moment of loss of stability, referred to the principal directions; and p is the external pressure.

From the equilibrium condition for an element of the shell, the following relation between σ_1, σ_2, and p is readily obtained:

$$\sigma_1/R_1 + \sigma_2/R_2 = p.$$

Taking this into account, we transform inequality $(*)$ into the form

$$\frac{2E}{\sqrt{3}(1-\nu^2)R_1R_2} \leq \left\{ 4p^2 + \frac{16\delta^2}{R_1R_2}\,(\tau^2 - \sigma_1\sigma_2) \right\}^{1/2} + p. \tag{**}$$

The expression $\tau^2 - \sigma_1\sigma_2$ is invariant with respect to orthogonal transformations at the given point and, consequently, is equal to $-\overline{\sigma}_1\overline{\sigma}_2$, where $\overline{\sigma}_1$ and $\overline{\sigma}_2$ are the principal stresses.

Let us now suppose that, at the moment of loss of stability of the shell, the stresses σ in all sections of the middle surface are compressive. Then $\overline{\sigma}_1$ and $\overline{\sigma}_2$ have the same sign and, consequently, $\tau^2 - \sigma_1\sigma_2 \leq 0$. Under this assumption, inequality $(**)$ is strengthened if the term $\tau^2 - \sigma_1\sigma_2$ is omitted from its right-hand side. We then obtain

$$\frac{1}{3}\,\frac{2E\delta^2}{\sqrt{3}(1-\nu^2)R_1R_2} \leq p. \tag{***}$$

Let now the shell be fixed in some manner along its edge and be under external pressure p. At a certain value of p it loses stability and begins to buckle. We weaken the fixing of the edge to the fixing conditions of the shell considered above and apply the distributed forces caused by the initial fixing. In this case our shell will be in the same conditions as the one considered. Since, by assumption, the normal stresses in the middle surface are compressive in all sections, the critical external pressure satisfies inequality $(***)$, which gives the required estimate for it.

Thus, if in the middle surface of a strictly convex shell under external pressure the normal stresses in all directions at the moment of loss of stability are compressive, then the critical pressure p $(***)$, at which loss of stability occurs, is independent of the nature of the imperfection of the edge fixing.

Consider, as an example, a spherical segment of radius R. In this case $p > 0.4E(\delta/R)^2$, i.e., the critical external pressure amounts to at least one third of the upper critical pressure corresponding to perfect clamping ($p_k \cong 1.2E\delta^2/R^2$).

The foregoing gives grounds for adopting the pressure

$$p^* = \frac{1}{3}\,\frac{2E\delta^2}{\sqrt{3}(1-\nu^2)R_1R_2}$$

as the design pressure in shell design.

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CITED LITERATURE

1. A. V. Pogorelov, *Geometric Methods in the Nonlinear Theory of Shells*, “Nauka,” 1967. A. V. Pogorelov, *Geometric Theory of Stability of Shells*, “Nauka,” 1966.

Note: Figure translations are in progress. See original paper for figures.