ON THE POSSIBILITY OF STABILIZING THE FLUTE INSTABILITY OF A PLASMA BY MEANS OF A FEEDBACK SYSTEM

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Fig. 1

Figure 1: Fig. 1

Abstract

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ON THE POSSIBILITY OF STABILIZING THE FLUTE INSTABILITY OF A PLASMA BY MEANS OF A FEEDBACK SYSTEM

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1. It is known that in adiabatic traps with a magnetic field decreasing away from the axis, a plasma of sufficiently high density is unstable with respect to flute perturbations elongated along the lines of force of the magnetic field. The instability is due to charge separation in the tongue of the perturbation (Fig. 1*a*), associated with the difference between the velocities of the azimuthal drift of electrons and ions in an inhomogeneous magnetic field. The azimuthal electric field E_θ that arises in this case leads to drift of the entire tongue outward from the plasma, i.e., to further growth of the perturbation. The field E_θ causing the instability is weakened by charges induced in the metallic wall enclosing the plasma of the chamber (Fig. 1*b*). The action of these charges leads to the fact that the density stability boundary lies above the corresponding value for the case of a free cylinder. The influence of the walls affects stability if they are located at a distance from the plasma less than, or of the order of, the wavelength of the perturbation.

Fig. 1

In the present work we consider the case in which, between the plasma and the chamber wall, there is a surface whose potential, by means of an external radio-engineering system (sensors and amplifiers), is maintained proportional to the plasma potential. The parameters of the radio-engineering circuit can be chosen so that, taking into account the field of the charges located on this surface, the resulting azimuthal electric field has the sign opposite to that which is obtained because of separation of plasma charges (Fig. 1*c*). The drift of the plasma in such a field reduces the initial perturbation, i.e., stabilization takes place.

Mathematically, the difference between the formulation of the problem of oscillations of a plasma surrounded by an "active" surface and the usual one reduces to replacing boundary conditions of the type $\varphi(\infty) = 0$ (or the vanishing of the potential on the chamber walls) by an additional condition (2).

2. Let us consider the stability of an infinite cylinder of rarefied $(\beta = 8\pi nT/H_0^2 \ll 1)$ inhomogeneous plasma of radius a, with its axis directed along an external homogeneous magnetic field H_0, with respect to potential flute perturbations $\psi = \varphi(r)\exp(im\theta - i\omega t)$, where θ–

azimuthal angle. The effect of the curvature of the magnetic-field lines, which occurs in real traps and leads to flute instability, is taken into account by introducing an effective radial force of gravity with acceleration gr/a ($g = \text{const}$), in which the ions undergo an azimuthal drift with angular velocity $\omega^* = g/\omega_{Hi}a$, where $\omega_{Hi} = eH_0/m_i c$ is the ion cyclotron frequency. In the absence of constant electric fields and neglecting the finiteness of the ion Larmor radius, the equation for flute oscillations has the form [1–3]

$$\frac{1}{r}\frac{d}{dr}\left[r^3 S\frac{d}{dr}\left(\frac{\varphi}{r}\right)\right]+\left[(1-m^2)\frac{S}{r}+\frac{m\omega_{0i}^2}{\omega_{Hi}}\left(\frac{1}{\omega}-\frac{1}{\omega+m\omega^*}\right)\frac{dN}{dr}+\frac{\omega_{0i}^2}{\omega_{Hi}^2}\frac{dN}{dr}\right]\varphi=0, \tag{1}$$

where $S = 1 + \dfrac{\omega_{0i}^2}{\omega_{Hi}^2}N(r)$; $N = n/n_0$; $n(r)$ is the plasma density; n_0 is the value of the density on the axis; $\omega_{0i} = (4\pi e^2 n_0/m_i)^{1/2}$ is the ion plasma frequency. As the boundary condition on the surface of radius b surrounding the plasma we shall take

$$\varphi(b) = \delta\varphi(a), \tag{2}$$

where δ is a coefficient determined by the external radio-engineering system (in general, depending on ω). The case of a grounded metallic wall corresponds to $\delta = 0$. It is known [4] that in this case the plasma is unstable beginning with densities for which $\omega_{0i}^2 \sim \omega_{Hi}\omega^*$. We shall show that, by a suitable choice of δ, one can substantially raise the instability threshold.

Let the plasma have a sharp boundary: $N(r) = 1$ for $r < a$, $N(r) = 0$ for $r > a$. Then the solution of Eq. (1) has the form $\varphi = A(r/a)^{|m|}$ for $r < a$; $\varphi = B(r/a)^{|m|} + C(a/r)^{|m|}$ for $r > a$. Using (2), the continuity of φ at the point $r = a$, and the condition

$$a^2\left[\frac{d}{dr}\left(\frac{\varphi}{r}\right)\Big|_{r=a+0}-\left(1+\frac{\omega_{0i}^2}{\omega_{Hi}^2}\right)\frac{d}{dr}\left(\frac{\varphi}{r}\right)\Big|_{r=a-0}\right]=$$

$$=\frac{m\omega_{0i}^2}{\omega_{Hi}}\left(\frac{1}{\omega}-\frac{1}{\omega+m\omega^*}\right)\varphi(a)+\frac{\omega_{0i}^2}{\omega_{Hi}^2}\varphi(a),$$

obtained by integrating (1) over the infinitely thin layer $a-\varepsilon < r < a+\varepsilon$, it is easy to obtain the equation for the frequency:

$$\frac{\delta - (b/a)^{|m|}}{(b/a)^{|m|} - (a/b)^{|m|}} = \frac{\omega_{0i}^2}{2\omega_{Hi}^2} + \frac{m\omega_{0i}^2}{2|m|\omega_{Hi}} \left(\frac{1}{\omega} - \frac{1}{\omega + m\omega^*} \right). \tag{3}$$

Let δ be real and independent of frequency. Denote the left-hand side of (3) by Λ. Taking into account that $\omega_{Hi} \gg \omega^*$, we obtain the necessary conditions for instability:

$$-\frac{2\omega_{0i}^2}{|m|\omega_{Hi}\omega^*} < \Lambda(\delta) < \frac{\omega_{0i}^2}{2\omega_{Hi}^2}. \tag{4}$$

For $\Lambda < 0$, in particular for $\delta = 0$ (metallic wall), the boundary of the stability region in density is determined by the left inequality in (4). The plasma is stable with respect to a perturbation of mode m if $\omega_{0i}^2 < \frac{1}{2}|m\Lambda|\omega_{Hi}\omega^*$. For $\Lambda > 0$, stability is determined by the right inequality. The oscillations are stable if

$$\delta > \left(\frac{b}{a}\right)^{|m|} + \frac{\omega_{0i}^2}{2\omega_{Hi}^2} \left[\left(\frac{b}{a}\right)^{|m|} - \left(\frac{a}{b}\right)^{|m|} \right]. \tag{5}$$

Let us now suppose that δ is complex, and find the conditions under which stable oscillations become damped. Let $\delta = \delta_0 + i\delta_1$, where δ_0 and δ_1 are real, δ_0 satisfies inequality (5), and $\delta_1/\delta_0 \ll 1$. For damping it is necessary that $\operatorname{Im}\omega < 0$. To first order in $\operatorname{Im}\delta$, the imaginary correction to the frequency is

$$\operatorname{Im}\omega = \frac{2|m|\omega_{Hi} \dfrac{d\Lambda}{d\delta} \operatorname{Im}\delta}{m\omega_{0i}^2 \dfrac{d}{d\omega} \left(\frac{1}{\omega} - \frac{1}{\omega + m\omega^*} \right)}.$$

It follows from this that the oscillations are damped if $\delta_1 \omega > 0$.

3. In the case of a sharp boundary considered above, the flute perturbations had the form of surface waves. If the boundary is diffuse, there exist oscillations localized inside the plasma and only weakly sensitive to the boundary conditions on the surface $r = b$. However, these perturbations, as well as higher azimuthal modes, can be stabilized by the finite Larmor radius ([5,1,6]). Of greatest interest is the behavior of the mode $|m| = 1$, which is not stabilized by the finite Larmor radius and for which $\varphi \sim r$. It is not difficult to show that, for $\omega_{0i}^2/\omega_{Hi}^2 \ll 1$ and a parabolic distribution of the plasma density over the radius, the stabilization criterion for this mode coincides with (5).

In the hydrodynamic approximation, the problem of plasma stabilization by feedbacks was considered in ([7]).

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Note: Figure translations are in progress. See original paper for figures.