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Abstract

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PHYSICS

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THEORY OF LASER GENERATION BY CENTERS OF TWO KINDS

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1. The most important problem of laser radiation is the problem of multimode generation. One of the causes of multimode behavior is the spectral inhomogeneity of the active medium. The simplest spectrally inhomogeneous system is a system of centers of two kinds. Attempts at a theoretical investigation of generation in such a system were undertaken in ([1,2]), where only individual problems were considered. A number of basic questions have not yet been resolved and, consequently, a general physical picture has not been obtained. In the present communication a closed theory of stationary generation in a system of 4-level centers of two kinds is given.

2. The kinetic equations describing laser generation in such a system have the form

$$dq^j/dt = [B_1(\omega^j)n_1 + B_2(\omega^j)n_2 - \gamma^j]q^j,$$

$$dn_k/dt = P_k - \Gamma_k n_k - \sum_j B_k(\omega^j)q^j n_k.$$

Here j is the number of the generating mode of the resonator, k is the kind of centers ($k = 1, 2$); q^j are the numbers of quanta in the modes; ω^j are the frequencies of the modes; γ^j are the mode losses; n_k are the populations of the upper working levels of the centers; $B_k(\omega)$ are the luminescence-line curves; P_k are the pump rates of the centers; Γ_k are the inverse lifetimes of the excited states of the centers with respect to spontaneous and nonradiative transitions.

We shall consider the most relevant case, when the type of modes capable of generating is fixed, and the discreteness step of the frequency of these modes is small in comparison with the width of the luminescence line of a center. We shall take $\gamma(\omega) = \gamma = \text{const}$. Finally, for simplicity, we shall assume that the curves $B_1(\omega)$, $B_2(\omega)$ are mirror images of one another with respect to $\bar{\omega} = (\omega_1 + \omega_2)/2$, where ω_1, ω_2 are the frequencies of the maxima of the luminescence lines.

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In the system under consideration either a single-mode or a two-mode regime can be realized. However, it is not known in advance which regions of parameters correspond to each of these regimes and, consequently, by what number of equations and unknowns the system is described. If the regime is single-mode, then there are 4 equations and unknowns: the frequency of the generating mode ω^0, the number of quanta in the mode q^0, and the populations n_1, n_2. If the regime is two-mode, then there are 6 equations and unknowns: two generation frequencies ω^1, ω^2, the corresponding numbers of quanta in the modes q^1, q^2, and the populations n_1, n_2. Therefore the method of solution will consist of the following. We solve the system of equations corresponding to each of the regimes. Then, from the additional conditions of positivity of the numbers of quanta and of maximality of the gain at the generation frequencies, we find which regions of parameters correspond to the given regime. Obviously, it is sufficient to consider the frequency interval (ω_1, ω_2).

3. The stationary single-mode regime is described by the system of equations:

$$B_1(\omega^0)n_1 + B_2(\omega^0)n_2 - \gamma = 0, \tag{1}$$

$$B_1'(\omega^0)n_1 + B_2'(\omega^0)n_2 = 0, \tag{2}$$

$$P_k - B_k(\omega^0)q^0 n_k - \Gamma_k n_k = 0, \qquad k = 1, 2, \tag{3}$$

under the additional conditions

$$B_1(\omega)n_1 + B_2(\omega)n_2 - \gamma < 0, \qquad \omega \neq \omega^0, \tag{4}$$

$$q^0 > 0. \tag{5}$$

Primes denote differentiation with respect to frequency.

Equation (2) means that the gain coefficient at the generation frequency has an extremum, and condition (4) means that this extremum is an absolute maximum.

Let us first consider (1)–(3) without taking (4), (5) into account. Then in (1)–(3), instead of ω^0, which is the generation frequency, there will appear a quantity having a more general meaning. We denote it by Ω and call it the fundamental parameter. In this case

$$n_1(\Omega) = \frac{\gamma}{B_1(\Omega) + B_2(\Omega)\left|B_1'(\Omega)/B_2'(\Omega)\right|}, \qquad n_2(\Omega) = \frac{\gamma}{B_2(\Omega) + B_1(\Omega)\left|B_2'(\Omega)/B_1'(\Omega)\right|}; \tag{6}$$

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$$q(\Omega) = \frac{P_1 - P_{1\text{por}}}{B_1(\Omega)n_1(\Omega)} = \frac{P_2 - P_{2\text{por}}}{B_2(\Omega)n_2(\Omega)}, \tag{7}$$

$$P_{1\text{por}} = \Gamma_1 n_1(\Omega), \qquad P_{2\text{por}} = \Gamma_2 n_2(\Omega).$$

Now let us take the additional conditions into account. Formulas (6) make it possible to determine in which frequency intervals (4) is satisfied. From (6) it is seen that $n_1 > n_2$ in the intervals $(\omega_1, \omega_{\text{I}})$, $(\bar{\omega}, \omega_{\text{II}})$, and $n_1 < n_2$ in the intervals $(\omega_{\text{I}}, \bar{\omega})$, $(\omega_{\text{II}}, \omega_2)$, where $\omega_{\text{I}}, \bar{\omega}, \omega_{\text{II}}$ are the roots of the equation

$$B_2'(\omega) + B_2'(\omega) = 0, \qquad \omega_{\text{I}} < \bar{\omega} < \omega_{\text{II}}.$$

Condition (4) for $\omega < \bar{\omega}$ is satisfied if $n_1 > n_2$; for $\omega > \bar{\omega}$, if $n_2 > n_1$. Therefore the single-mode regime is possible only if Ω lies in one of the intervals $(\omega_1, \omega_{\text{I}})$, $(\omega_{\text{II}}, \omega_2)$. In this case

$$\omega^0 = \Omega. \tag{8}$$

When (8) is fulfilled, the quantities $P_{1\text{por}}$ and $P_{2\text{por}}$ appearing in (7) are threshold pumpings.

4. The stationary two-mode regime is described by the system of equations with unknowns ω^1, ω^2 ($\omega^1 < \omega^2$), q^1, q^2, n_1, n_2:

$$B_1(\omega^j)n_1 + B_2(\omega^j)n_2 - \gamma = 0, \qquad j = 1, 2, \tag{9}$$

$$B_1'(\omega^j)n_1 + B_2'(\omega^j)n_2 = 0, \qquad j = 1, 2, \tag{10}$$

$$P_k - \left[B_k(\omega^1)q^1 + B_k(\omega^2)q^2\right] n_k - \Gamma_k n_k = 0, \qquad k = 1, 2, \tag{11}$$

under the additional conditions

$$B_1(\omega)n_1 + B_3(\omega)n_2 - \gamma < 0, \qquad \omega \neq \omega^1, \omega^2, \tag{12}$$

$$q^j > 0, \qquad j = 1, 2. \tag{13}$$

The solution of the system (9)–(11) is

$$\omega^1 = \omega_{\text{I}}, \qquad \omega^2 = \omega_{\text{II}};$$

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Fig. 1

Figure 1: Fig. 1

$$n_1 = n_2 = n \equiv \frac{\gamma}{B_{\rm I} + B_{\rm II}}, \qquad B_{\rm I} = B_1(\omega_{\rm I}) = B_2(\omega_{\rm II}),$$

$$B_{\rm II} = B_1(\omega_{\rm II}) = B_2(\omega_{\rm I});$$

$$q^1 = \Delta_1/\Delta, \qquad q^2 = \Delta_2/\Delta, \qquad \Delta_1 = \left[B_{\rm I}(P_1 - P_{1D}) - B_{\rm II}(P_2 - P_{2D})\right]/n,$$

$$\Delta_2 = \left[B_{\rm I}(P_2 - P_{2D}) - B_{\rm II}(P_1 - P_{1D})\right]/n, \tag{14}$$

$$\Delta = B_{\rm I}^2 - B_{\rm II}^2, \qquad P_{1D} = \Gamma_1 n, \qquad P_{2D} = \Gamma_2 n.$$

The generation frequencies ω^1, ω^2 depend only on the luminescence curves; n_1, n_2 depend on these curves and on γ; q^1, q^2 depend on the curves and on the quantities $P_1, P_2, \Gamma_1, \Gamma_2, \gamma$.

It is easy to see that condition (12) is satisfied automatically. From (13) it follows that the two-mode regime occurs if the fundamental parameter Ω lies in the interval $(\omega_{\rm I}, \omega_{\rm II})$. From (13) and (14) it follows that P_{1D} and P_{2D} are the threshold pumpings for the two-mode regime.

5. We can now obtain the complete picture of generation. From the preceding it is clear that the range of values of the fundamental parameter Ω for two-mode generation lies between two regions corresponding to one-mode generation. The dependences $n_1(\Omega), n_2(\Omega)$ over the whole interval (ω_1, ω_2) are shown in Fig. 1. From (7) it is seen that the curves in Fig. 1 also give the dependences $P_{1\rm por}/\Gamma_1$ and $P_{2\rm por}/\Gamma_2$ on the generation frequency in the regions where the one-mode regime exists. Formulas (7), (14) and Fig. 1 make it possible to obtain a clear picture of the dependence of the generation regimes on the pumpings of centers of both kinds and of the dependence of the radiation of the generating modes on these pumpings. This picture is schematically shown in Fig. 2. The pumpings P_1, P_2 are plotted on the coordinate axes. The region O_1 is the region of one-mode generation corresponding to the interval $(\omega_1, \omega_{\rm I})$, the region O_2 to the interval $(\omega_{\rm II}, \omega_2)$; D is the region of two-mode generation; π is the subthreshold region. The curves π_1, π_2 are the threshold curves for the regions O_1, O_2. They are obtained by eliminating Ω from (6), (7).

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Fig. 2

Figure 2: Fig. 2

Fig. 1

Fig. 2

The coordinates of the point $\pi_D(P_{1D}, P_{2D})$ give the threshold values of the pumpings for the two-mode regime. Rays in the regions O_1, O_2 are isochromes, i.e., lines of constant generation frequency (constant n_1, n_2). On the corresponding isochrome the relation between P_1 and P_2 is linear, but P_1 and P_2 are certainly not proportional to one another. Lines 1 and 2, which are respectively the boundaries $O_1 - D$ and $D - O_2$, give such a relation for the frequencies ω_{I} and ω_{II}.

The picture obtained makes it easy to trace the dependence of the change in the character of generation on changes in the pumpings. Let us consider several typical examples. Let $P_2 = \text{const}$. Then, if $P_2 < P_{2D}$, only one-mode generation is possible. As the pumping P_1 is increased, the generation frequency approaches ω_1. If $P_2 > P_{2D}$, then as P_1 is increased, one-mode generation first appears in the region O_2, then two-mode generation, and finally one-mode generation in O_1. For $P_1 = \text{const}$ and increasing P_2, everything occurs analogously.

Let now

$$P_1/P_2 = \text{const}, \tag{15}$$

i.e., let the pumpings be proportional to one another. To carry out the investigation, we proceed as follows. From the origin draw two rays 1 and 2, parallel to the straight lines 1 and 2. In this case two possibilities arise: a) the point π_D lies inside the sector $(\overline{1}, \overline{2})$ (Fig. 2a); b) the point π_D lies outside it (Fig. 2b). In Fig. 2b draw through the origin and the point π_D the straight line $\overline{3}$. If the straight line (15) lies inside the sector $(\overline{1}, \overline{2})$, then in both cases, as the pumpings are increased, a single-mode regime first arises and then a two-mode regime. Let now the straight line (15) lie outside the sector $(\overline{1}, \overline{2})$. Then in case a) only a single-mode regime is possible. In case b) the situation is as follows: if the straight line (15) lies outside the sector $(\overline{1}, \overline{3})$, only a single-mode regime is possible; if it lies inside the sector $(\overline{1}, \overline{3})$, then first a single-mode regime arises in one of the regions, then a two-mode regime, and finally a single-mode regime in the other region. The last case was observed experimentally ([3]). We note that if, for limiting strong pumpings, the generation is single-mode, then the limiting frequency is determined by that isochrome which is parallel to the straight line (15).

The picture obtained makes it possible to determine in which region generation occurs for given pumpings P_1, P_2, and, for the single-mode regime, to determine

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graphically the generation frequency ω^0 by means of the isochromes, if the parameters $\Gamma_1, \Gamma_2, \gamma$ are specified numerically and the curves $B_1(\omega), B_2(\omega)$ are concretized.

Finally, the picture obtained makes it possible to determine graphically the dependence of the radiation on the pumpings. From (7) it is seen that for the single-mode regime

$$\gamma q^0 = (P_1 - P_{1\text{por}}) + (P_2 - P_{2\text{por}}).$$

From (14) it is seen that for the two-mode regime

$$\gamma(q^1 + q^2) = (P_1 - P_{1D}) + (P_2 - P_{2D}),$$

and the ratio of the sought quantities is

$$q^1/q^2 = l/m,$$

where the segments l, m are laid off in Fig. 2. The construction of the segments corresponding to γq^0 in the case of the single-mode regime and to $\gamma q^1, \gamma q^2$ in the case of the two-mode regime is now obvious.

Let us dwell on the role of the distance between the maxima of the luminescence lines $\omega_2 - \omega_1$. With an increase of this quantity the rays 1 and 2 diverge, becoming, in the limit, parallel to the coordinate axes. The threshold curves in the limit turn into segments of straight lines parallel to the axes. This corresponds to independent centers. With a decrease of $\omega_2 - \omega_1$ the angle between the rays decreases, becoming equal to zero at $\omega_{\text{I}} = \overline{\omega} = \omega_{\text{II}}$, or

$$B_k''(\overline{\omega}) = 0, \qquad k = 1, 2. \tag{16}$$

For values of $\omega_2 - \omega_1$ corresponding to (16) and smaller, only a single-mode regime is possible.

In conclusion, we note that abandoning the assumption of identical shapes of the curves $B_1(\omega)$ and $B_2(\omega)$ leads only to quantitative changes; the character of the picture obtained does not change.

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